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APPROXIMATION OF A COMMON FIXED POINT IN A GEODESIC SPACE WITH CURVATURE BOUNDED ABOVE

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Dedicated to Professor Wataru Takahashi on his 70th birthday

Abstract. We propose an iterative method generated by the shrinking projection method for approximating a common fixed point of a finite family of nonexpansive mappings defined on a complete geodesic space. The main result shows that, even if the sequence of errors for obtaining the value of metric projections does not converge to zero, the generated sequence still has an appropriate property.

1. Introduction

Fixed point problem for a nonexpansive mapping is one of the most crucial problems in nonlinear analysis and it can be applied to many other problems such as convex minimization problems, variational inequality problems, equilibrium problems, minimax problems, and others. Furthermore, common fixed problems for a family of nonexpansive mappings have been also studied by a large number of researchers as a generalization of fixed point problem for a single mapping.

An approximation of a solution to the common fixed point problem for nonexpansive mappings is formulated as to find a sequence converging to a point $z \in X$ such that $z = T_i z$ for every $i \in I$, where $\{T_i\}$ is a given family of nonexpansive mappings defined on a metric space X.

The following result, which is called the shrinking projection method, was first proposed by Takahashi, Takeuchi, and Kubota [14] as an iterative scheme to approximate a common fixed point of nonexpansive mappings.

Theorem 1.1 (Takahashi, Takeuchi, and Kubota [14]). *Let H be a Hilbert space and C a nonempty closed convex subset of H. Let $\{T_\lambda : \lambda \in \Lambda\}$ be a family of nonexpansive mappings of C into itself and $\{S_n\}$ a sequence of nonexpansive mappings of C into itself satisfying $\emptyset \neq \bigcap_{\lambda\in\Lambda} F(T_\lambda) \subset \bigcap_{n=1}^{\infty} F(S_n)$. Suppose that $\{S_n\}$ satisfies the NST condition (I) with $\{T_\lambda\}$. Let $\{\alpha_n\}$ be a sequence in $[0, a]$, where $0 < a < 1$. For an arbitrary point $x \in H$, generate a sequence $\{x_n\}$ by the following iterative scheme: $x_1 \in C$, $C_1 = C$, and*

$$\begin{aligned} y_n &= \alpha_n x_n + (1-\alpha_n) S_n x_n, \\ C_{n+1} &= \{z \in C_n : \|z - y_n\| \leq \|z - x_n\|\}, \\ x_{n+1} &= P_{C_{n+1}} x \end{aligned}$$

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for $n \in \mathbb{N}$. Then, $\{x_n\}$ converges strongly to $P_F x \in C$, where $F = \bigcap_{\lambda \in \Lambda} F(T_\lambda)$ and P_K is the metric projection of H onto a nonempty closed convex subset K of H.

After this theorem was released, a number of generalized results have been proposed; for instance, see Takahashi and Zembayashi [15], Plubtieng and Ungchittrakool [11], Inoue, Takahashi, and Zembayashi [3], Qin, Cho, and Kang [12], Wattanawitoon and Kumam [16, 17], Kimura, Nakajo, and Takahashi [7], Kimura and Takahashi [10], and others. These results considered the case where the underlying space is a Banach space having certain additional properties.

On the other hand, fixed point theory on a complete metric space with convexity structures was firstly studied by Takahashi [13] and has been investigated from various aspects; see also [1, 2]. The shrinking projection method on a geodesic space was firstly considered by Kimura [5] for the case of a real Hilbert ball. The following result is for the case of a subset of the unit sphere of a Hilbert space proved by Kimura and Satô [9].

Theorem 1.2 (Kimura and Satô [9])**.** *Let S_H be the unit sphere of a real Hilbert space H with the metric d defined by a length of minimal great arc, and C a closed convex subset of S_H such that $d(u, v) < \pi/2$ for every $u, v \in C$. Let $T : C \to C$ be a nonexpansive mapping such that the set of fixed points $F = \{z \in C : Tz = z\}$ is nonempty. For a given initial point $x_0 \in C$ and $C_0 = C$, generate a sequence $\{x_n\}$ as follows:*

$$C_{n+1} = \{z \in C : d(Tx_n, z) \le d(x_n, z)\} \cap C_n,$$
$$x_{n+1} = P_{C_{n+1}} x_0,$$

for each $n \in \mathbb{N}$. Then $\{x_n\}$ is well defined and converges to $P_F x_0 \in C$, where $P_K : C \to K$ is the metric projection of C onto a nonempty closed convex subset K of C.

In this paper, we deal with an approximation of common fixed points of a finite family of nonexpansive mappings defined on a complete CAT(1) space with calculation errors. The main result shows that, even if the sequence of errors for obtaining the value of metric projections has a positive upper limit, the generated sequence still has a nice property for approximating a common fixed point of the mappings. To prove the results, we employ the technique used in [4, 6].

2. Preliminaries

Let X be a π-geodesic metric space, that is, for every two points in X having the metric between them less than π, there exists a geodesic connecting them. Suppose that X is π-uniquely geodesic, that is, each geodesic connecting u and v with $d(u, v) < \pi$ is uniquely determined. Then, for every $u, v \in X$ with $d(u, v) < \pi$ and for $t \in [0, 1]$, a point $w \in X$ such that $d(w, v) = td(u, v)$ and $d(u, w) = (1 - t)d(u, v)$ is also unique. We denote this point w by $tu \oplus (1 - t)v$. In a geodesic space, we can define the convexity of subsets of X in a natural way. For the detail, see [1].

We say X is a CAT(1) space if for each geodesic triangle on X is as thin as its comparison triangle on the 2-dimensional unit sphere $\mathbb{S}^2$. To be precise, every

$p, q \in \triangle \subset X$ and their comparison points $\overline{p}, \overline{q} \in \overline{\triangle} \subset \mathbb{S}^2$ satisfy the CAT(1) inequality

$$d(p,q) \le d_{\mathbb{S}^2}(\overline{p}, \overline{q}),$$

where $d_{\mathbb{S}^2}$ is the spherical metric defined on $\mathbb{S}^2$. We know that if X is a CAT(1) space, then for $x, y, z \in X$ such that $d(y,z) + d(z,x) + d(x,y) < 2\pi$ and $t \in [0,1]$, the following holds [8]:

$$\sin d(x,y) \cos d(tx \oplus (1-t)y, z) \\ \ge \sin(td(x,y)) \cos d(x,z) + \sin((1-t)d(x,y)) \cos d(y,z).$$

This inequality plays an important role in our result.

Let X be a complete CAT(1) space such that $d(u,v) < \pi/2$ for every $u, v \in X$, and C a nonempty closed convex subset C of X. We know that for every $x \in X$, there exists a unique $y_x \in C$ such that $d(x, y_x) = d(x, C)$, where $d(x,C) = \inf_{y \in C} d(x,y)$. We define a mapping $P_C : X \to C$ by $P_C x = y_x$ for $x \in X$ and we call it the metric projection of X onto C.

The following lemma is also obtained from the result in [8].

Lemma 2.1 (Kimura and Satô [8]). *Let X be a complete* CAT(1) *space such that $d(u,v) < \pi/2$ for every $u, v \in X$. Let $\{C_n\}$ be a sequence of nonempty closed convex subsets of X such that $C_{n+1} \subset C_n$ for every $n \in \mathbb{N}$ and $C_0 = \bigcap_{n=1}^{\infty} C_n \neq \emptyset$. Let $\{P_{C_n}\}$ be a sequence of metric projections corresponding to $\{C_n\}$. Then, for $u \in X$, a sequence $\{P_{C_n} u\}$ converges to $P_{C_0} u \in X$, where P_{C_0} is a metric projection of X onto C_0.*

A mapping T of a metric space X to itself is said to be nonexpansive if $d(Tx, Ty) \le d(x,y)$ for every $x, y \in X$. The set of all fixed points of T is denoted by $F(T)$. We know that if X is CAT(1) space with $d(u,v) < \pi$ for every $u, v \in X$, then $F(T)$ is closed and convex.

3. Shrinking projection method with errors

In this section, we show that the iterative sequence generated by the shrinking projection method with calculation errors has a certain appropriate property for approximating a solution to the common fixed point problem even if the upper limit of the error sequence is a positive value.

Theorem 3.1. *Let X be a complete* CAT(1) *space such that $d(u,v) < \pi/2$ for every $u, v \in X$ and that a subset $\{z \in X : d(v,z) \le d(u,z)\}$ is convex for every $u, v \in X$. Let $\{T_j : j = 0, 1, \dots, k-1\}$ be a family of nonexpansive mappings such that the set of their common fixed point $F = \bigcap_{j=0}^{k-1} F(T_j)$ is nonempty. Let $\{\epsilon_n\}$ be a sequence in $[0, \infty[$ and let $\epsilon_0 = \limsup_{n\to\infty} \epsilon_n$. For a given point $u \in X$, generate a sequence $\{x_n\}$ as follows: $x_1 = u$, $C_1 = X$, and*

$$C_{n+1} = \{z \in X : d(T_{(n \bmod k)} x_n, z) \le d(x_n, z)\} \cap C_n,$$
$$x_{n+1} \in C_{n+1} \text{ such that } \cos d(u, x_{n+1}) \ge \cos d(u, C_{n+1}) \cos \epsilon_{n+1},$$

for each $n \in \mathbb{N}$. Then

$$\limsup_{n\to\infty} d(x_n, T_j x_n) \le 4\epsilon_0$$

for every $j \in \{0, 1, \ldots, k-1\}$. *Moreover, if* $\epsilon_0 = 0$, *then* $\{x_n\}$ *converges to* $P_F x_0$, *where* P_F *is the metric projection of* X *onto* F.

Proof. First we prove that $C_n \neq \emptyset$ for every $n \in \mathbb{N}$, which implies that $\{x_n\}$ is well defined. Since each T_j is nonexpansive, we have that $d(T_j x, z) \leq d(x, z)$ for every $x \in X$, $z \in F$, and $j = 0, 1, \ldots, k-1$. It follows that $F \subset C_n$ for all $n \in \mathbb{N}$ and by assumption, F is nonempty and so is C_n. Hence the sequence $\{x_n\}$ is well defined. We also know that C_n is closed and convex for every $n \in \mathbb{N}$. Indeed, it is trivial that C_n being closed. The convexity of C_n is obtained from the assumption of the space. Therefore, we can define the metric projection P_{C_n} of X onto C_n. Let $p_n = P_{C_n} u$ for all $n \in \mathbb{N}$. Then, by Lemma 2.1, $\{p_n\}$ converges to $p_0 = P_{C_0} u$, where $C_0 = \bigcap_{n=1}^{\infty} C_n$. Since $x_n \in C_n$ and $d(u, C_n) = d(u, p_n)$, we have that

$$\cos d(u, x_{n+1}) \geq \cos d(u, C_{n+1}) \cos \epsilon_{n+1}$$

for every $n \in \mathbb{N}$. Then we have that

$$\begin{aligned} &\sin d(p_n, x_n) \cos d(p_n, u) \\ &\geq \sin d(p_n, x_n) \cos d(\alpha p_n \oplus (1-\alpha) x_n, u) \\ &\geq \sin(\alpha d(p_n, x_n)) \cos d(p_n, u) + \sin((1-\alpha) d(p_n, x_n)) \cos d(x_n, u) \end{aligned}$$

for $\alpha \in]0, 1[$, and hence

$$\sin d(p_n, x_n) - \sin(\alpha d(p_n x_n)) \geq \sin((1-\alpha) d(p_n x_n)) \frac{\cos d(x_n, u)}{\cos d(p_n, u)}.$$

If $p_n \neq x_n$, then dividing by $2 \sin(\frac{1-\alpha}{2} d(p_n, x_n))$, we have that

$$\cos\left(\frac{1+\alpha}{2} d(p_n, x_n)\right) \geq \cos\left(\frac{1-\alpha}{2} d(p_n, x_n)\right) \frac{\cos d(x_n, u)}{\cos d(p_n, u)}.$$

Notice that this inequality also holds even if $p_n = x_n$. Tending $\alpha \to 1$, we have that

$$\cos d(p_n, x_n) \geq \frac{\cos d(x_n, u)}{\cos d(p_n, u)} = \frac{\cos d(x_n, u)}{\cos d(u, C_n)} \geq \cos \epsilon_n,$$

that is,

$$d(p_n, x_n) \leq \epsilon_n$$

for every $n \in \mathbb{N}$. Since $p_n \in C_n$, we also get that

$$d(T_{(n \bmod k)} x_n, p_n) \leq d(x_n, p_n) \leq \epsilon_n$$

for every $n \in \mathbb{N}$.

Fix $j \in \{0, 1, \ldots, k-1\}$. For each $n \in \mathbb{N}$, there exists $i_n \in \{0, 1, \ldots, k-1\}$ such that

$$(n + i_n) \bmod k = j.$$

Then we have that

$$\begin{aligned} d(x_n, T_j x_n) &\leq d(x_n, p_{n+i_n}) + d(p_{n+i_n}, T_j x_{n+i_n}) + d(T_j x_{n+i_n}, T_j x_n) \\ &\leq d(x_n, p_{n+i_n}) + d(p_{n+i_n}, T_j x_{n+i_n}) + d(x_{n+i_n}, x_n) \\ &\leq d(x_n, p_{n+i_n}) + d(p_{n+i_n}, T_{((n+i_n) \bmod k)} x_{n+i_n}) + d(x_{n+i_n}, x_n) \\ &\leq d(x_n, p_{n+i_n}) + d(p_{n+i_n}, x_{n+i_n}) + d(x_{n+i_n}, x_n) \\ &\leq d(x_n, p_{n+i_n}) + \epsilon_{n+i_n} + d(x_{n+i_n}, x_n). \end{aligned}$$

We also have that

$$d(x_n, p_{n+i_n}) \le d(x_n, p_n) + d(p_n, p_{n+i_n}) \le \epsilon_n + d(p_n, p_{n+i_n})$$

and

$$\begin{aligned} d(x_{n+i_n}, x_n) &\le d(x_{n+i_n}, p_{n+i_n}) + d(p_{n+i_n}, p_n) + d(p_n, x_n) \\ &\le \epsilon_{n+i_n} + \epsilon_n + d(p_{n+i_n}, p_n). \end{aligned}$$

Thus it follows that

$$d(x_n, T_j x_n) \le 2(\epsilon_n + \epsilon_{n+i_n} + d(p_{n+i_n}, p_n))$$

for every $n \in \mathbb{N}$. Since

$$\limsup_{n\to\infty} \epsilon_{n+i_n} \le \limsup_{n\to\infty} \epsilon_n = \epsilon_0$$

and $\limsup_{n\to\infty} d(p_{n+i_n}, p_n) = 0$, we obtain that

$$\limsup_{n\to\infty} d(x_n, T_j x_n) \le 4\epsilon_0,$$

the desired result.

For the latter part of the theorem, suppose that $\epsilon_0 = 0$. Then we have that

$$\limsup_{n\to\infty} d(x_n, p_n) \le \limsup_{n\to\infty} \epsilon_n = 0.$$

It implies that $\lim_{n\to\infty} d(x_n, p_n) = 0$ and thus $\{x_n\}$ converges to $p_0 = P_{C_0} u$. Since

$$0 \le \liminf_{n\to\infty} d(x_n, T_j x_n) \le \limsup_{n\to\infty} d(x_n, T_j x_n) \le 4\epsilon_0 = 0,$$

we also have that $\{T_j x_n\}$ converges to p_0 for each $j \in \{0, 1, \dots, k-1\}$. By the continuity of the nonexpansive mapping T_j, we have that

$$T_j p_0 = T_j \left(\lim_{n\to\infty} x_n \right) = \lim_{n\to\infty} T_j x_n = p_0,$$

that is, $p_0 \in F = \bigcap_{j=0}^{k-1} F(T_j)$. Since $F \subset C_0$, we get that

$$p_0 = P_{C_0} u = P_F u,$$

which completes the proof. □

We can also prove the following result, which shows that each point of the iterative sequence can be obtained by the direct calculation of the distance among the set and the points if the diameter of the space is less than $\pi/2$.

Theorem 3.2. *Let X be a complete* CAT(1) *space such that $D = \operatorname{diam} X < \pi/2$ and that a subset $\{z \in X : d(v, z) \le d(u, z)\}$ is convex for every $u, v \in X$. Let $\{T_j : j = 0, 1, \dots, k-1\}$ be a family of nonexpansive mappings such that the set of their common fixed point $F = \bigcap_{j=0}^{k-1} F(T_j)$ is nonempty. Let $\{\delta_n\}$ be a sequence in $[0, \infty[$ and let $\delta_0 = \limsup_{n\to\infty} \delta_n$. For a given point $u \in X$, generate a sequence $\{x_n\}$ as follows: $x_1 = u$, $C_1 = X$, and*

$$\begin{aligned} &C_{n+1} = \{z \in X : d(T_{(n \bmod k)} x_n, z) \le d(x_n, z)\} \cap C_n, \\ &x_{n+1} \in C_{n+1} \text{ such that } d(u, x_{n+1}) \le d(u, C_{n+1}) + \delta_{n+1}, \end{aligned}$$

for each $n \in \mathbb{N}$*. Then*

$$\limsup_{n\to\infty} d(x_n, T_j x_n) \leq 4 \arccos(e^{-\delta_0 \tan D})$$

for every $j \in \{0, 1, \dots, k-1\}$*. Moreover, if* $\delta_0 = 0$*, then* $\{x_n\}$ *converges to* $P_F x_0$*, where* P_F *is the metric projection of* X *onto* F*.*

Proof. For $n \in \mathbb{N}$, let

$$\epsilon_n = \arccos(e^{-\delta_n \tan D}),$$

which is equivalent to that

$$\delta_n = -\frac{\log \cos \epsilon_n}{\tan D}.$$

Since it holds that $d(u, C_n) \leq d(u, x_n) \leq d(u, C_n) + \delta_n$, by applying the mean value theorem with the function $g(t) = -\log \cos t$, we can find $t_0 \in \mathbb{R}$ such that $d(u, C_n) \leq t_0 \leq d(u, x_n)$ and

$$g(d(u, x_n)) - g(d(u, C_n)) = g'(t_0)(d(u, x_n) - d(u, C_n)) \leq g'(t_0)\delta_n.$$

We also have that

$$g'(t_0) = \tan t_0 \leq \tan d(u, x_n) \leq \tan D,$$

which implies that

$$g(d(u, x_n)) - g(d(u, C_n)) \leq \delta_n \tan D = g(\epsilon_n).$$

It follows that

$$\begin{aligned} \log \cos(d(u, x_n)) &\geq \log \cos d(u, C_n) + \log \cos \epsilon_n \\ &= \log(\cos d(u, C_n) \cos \epsilon_n). \end{aligned}$$

Thus we obtain that $\cos d(u, x_n) \geq \cos d(u, C_n) \cos \epsilon_n$. This inequality shows that the previous theorem is applicable for this sequence and consequently we get that

$$\begin{aligned} \limsup_{n\to\infty} d(x_n, T_j x_n) &\leq 4 \limsup_{n\to\infty} \epsilon_n \\ &= 4 \limsup_{n\to\infty} \arccos(e^{-\delta_n \tan D}) \\ &= 4 \arccos(e^{-\delta_0 \tan D}) \end{aligned}$$

for each $j \in \{0, 1, \dots, k-1\}$. The remainder part of the theorem is also obtained from the previous theorem. □

In the end of this section, we remark several things about our main result. For known iterative schemes such as the Halpern type method and the Mann type method, the norms of error terms must be summable because the effect of error terms will accumulate as the iteration progresses. The main result shows that, in the proposed iterative scheme, we do not need to suppose the summability of error terms. This will be useful for practical calculation for computer simulations.

Our result shows that the inequality

$$\max_{j\in\{0,1,2,\dots,k-1\}} d(x_n, T_j x_n) < \epsilon$$

is available for the terminating condition for sufficiently small $\epsilon > 0$. This condition is simple and often used in the computer simiulations. However, we note that this condition does not guarantee that the iterative sequence $\{x_n\}$ certainly approaches to a common fixed point of $\{T_j\}$ in general. Consider the following mapping: Let $X = \mathbb{S}^2 \cap \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 : x_3 \geq \sqrt{3}/2\}$. Then, by using the polar coordinates, we can write

$$X = \{x = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \in \mathbb{R}^3 : 0 \leq \theta \leq \pi/6, 0 \leq \phi < 2\pi\}.$$

For a small $\epsilon > 0$, define $T : X \to X$ by

$$\begin{aligned} Tx &= T(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \\ &= \begin{cases} (\sin(\theta - \epsilon/2)\cos\phi, \sin(\theta - \epsilon/2)\sin\phi, \cos(\theta - \epsilon/2)) & (\theta > \epsilon/2) \\ (0, 0, 1) & (\theta \leq \epsilon/2) \end{cases} \end{aligned}$$

for $x \in X$. Then $d(x, Tx) \leq \epsilon/2 < \epsilon$ for all $x \in X$, whereas $F(T) = \{(0, 0, 1)\} \subset X$.

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