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EDELSTEIN'S FIXED POINT THEOREM IN GENERALIZED METRIC SPACES

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Dedicated to Professor Wataru Takahashi on the occasion of his 70th birthday

Abstract. We introduce the concept of compactness of generalized metric spaces. We also prove fixed point theorems which are generalizations of Edelstein's fixed point theorem.

1. Introduction

Throughout this paper we denote by $\mathbb{N}$ the set of all positive integers.

In 2000, Branciari in [2] introduced the following, very interesting concept. See also [4, 5] and others.

Definition 1.1 (Branciari [2])**.** Let X be a set, let d be a function from $X \times X$ into $[0, \infty)$ and let $\nu \in \mathbb{N}$. Then (X, d) is said to be a *ν-generalized metric space* if the following hold:

- (N1) $d(x, y) = 0$ iff $x = y$ for any $x, y \in X$.
- (N2) $d(x, y) = d(y, x)$ for any $x, y \in X$.
- (N3) $d(x, y) \leq d(x, u_1) + d(u_1, u_2) + \cdots + d(u_\nu, y)$ for any $x, u_1, u_2, \ldots, u_\nu, y \in X$ such that $x, u_1, u_2, \ldots, u_\nu, y$ are all different.

It is obvious that (X, d) is a metric space if and only if (X, d) is a 1-generalized metric space. Very recently, in [6], we found that not every generalized metric space has the compatible topology. See also [7]. In [1], we discussed the completeness of ν-generalized metric spaces.

Motivated by these, in this paper, we introduce the concept of compactness. We also prove fixed point theorems which are generalizations of Edelstein's fixed point theorem [3].

2. Preliminaries

As mentioned above, in general, ν-generalized metric spaces do not necessarily have the compatible topology. So we have to define the concept of the convergence.

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Definition 2.1. Let (X, d) be a ν-generalized metric space.

- A sequence $\{x_n\}$ in X is said to be *Cauchy* iff $\lim_n \sup_{m>n} d(x_m, x_n) = 0$ holds.
- A sequence $\{x_n\}$ in X is said to *converge* to x iff $\lim_n d(x, x_n) = 0$ holds.
- A sequence $\{x_n\}$ in X is said to *converge exclusively* to x iff $\lim_n d(x, x_n) = 0$ holds and $\lim_n d(y, x_{f(n)}) = 0$ does not hold for any $y \in X \setminus \{x\}$ and for any subsequence $\{x_{f(n)}\}$ of $\{x_n\}$.
- A sequence $\{x_n\}$ in X is said to *converge to x in the strong sense* iff $\{x_n\}$ is Cauchy and $\{x_n\}$ converges to x.

Definition 2.2. Let (X, d) be a ν-generalized metric space.

- X is *compact* iff for any sequence $\{x_n\}$ in X, there exists a subsequence $\{x_{f(n)}\}$ of $\{x_n\}$ converging to some $z \in X$.
- X is *compact in the strong sense* iff for any sequence $\{x_n\}$ in X, there exists a subsequence $\{x_{f(n)}\}$ of $\{x_n\}$ converging to some $z \in X$ in the strong sense.

Proposition 2.3. *Let (X, d) be a ν-generalized metric space and let $\{x_n\}$ be a sequence in X and let z be an element of X. Then the following hold*:

(i) *$\{x_n\}$ converges to z if and only if for every subsequence $\{x_{f(n)}\}$ of $\{x_n\}$, there exists a subsequence $\{x_{f(g(n))}\}$ of $\{x_{f(n)}\}$ converging to z.*

(ii) *If $\{x_n\}$ converges to z in the strong sense, then $\{x_n\}$ converges exclusively to z.*

(iii) *If $\{x_n\}$ converges to z, $\lim_n d(x_n, x_{n+1}) = 0$ holds and $x_n \neq z$ for any $n \in \mathbb{N}$, then $\{x_n\}$ is Cauchy, that is, $\{x_n\}$ converges to z in the strong sense.*

Proof. In the case where $\nu = 1$, the conclusions are obvious. So we assume $\nu \geq 2$. The classical proof works on (i). We next show (ii). Arguing by contradiction, we assume that there exists a subsequence $\{x_{f(n)}\}$ of $\{x_n\}$ converging to some $v \in X \setminus \{z\}$. Since $\{x_n\}$ converges to z, we may assume that $x_{f(n)} \neq v$ for $n \in \mathbb{N}$. Since $\{x_{f(n)}\}$ converges to v, we may also assume that $x_{f(n)} \neq z$ for $n \in \mathbb{N}$ and $x_{f(n)}$ are all different. We have

$$d(v, z) \leq \lim_{n \to \infty} \Big(d(v, x_{f(n)}) + \sum_{j=n}^{n+\nu-2} d(x_{f(j)}, x_{f(j+1)}) + d(x_{f(n+\nu-1)}, z) \Big) = 0,$$

which implies a contradiction. Therefore we have shown $\{x_n\}$ converges exclusively to z. Let us prove (iii). Noting $x_n \neq z$ for $n \in \mathbb{N}$, we can define a function h from $\mathbb{N}$ into itself by

$$h(n) := \max\{i : \ x_i = x_n\} < \infty.$$

Arguing by contradiction, we assume that $\{x_n\}$ is not Cauchy. Then there exist subsequences $\{x_{f(n)}\}$ and $\{x_{g(n)}\}$ of $\{x_n\}$ such that

$$\max\big\{f(n), \ g(n)\big\} < \min\big\{f(n+1), \ g(n+1)\big\}$$

and $d(x_{f(n)}, x_{g(n)}) \geq (\nu + 1)\,\varepsilon$ for some $\varepsilon > 0$. We choose $\ell \in \mathbb{N}$ such that

$$d(x_n, x_{n+1}) < \varepsilon \quad \text{and} \quad d(x_n, z) < \varepsilon$$

for $n \geq \ell$. Fix $m \in \mathbb{N}$ with $f(m) \geq \ell$. Without loss of generality, we may assume $h(f(m)) < h(g(m))$. Put $k_0 = h(g(m))$, $k_1 = h(k_0) + 1$, ..., $k_{\nu-1} = h(k_{\nu-2}) + 1$. We also put $y_i = x_{k_i}$ for $i = 0, 1, \ldots, \nu - 1$, $y_\nu = z$ and $y_{\nu+1} = x_{h(f(m))}$. Then y_i are all different. So we have

$$(\nu + 1)\,\varepsilon \leq d(x_{f(m)}, x_{g(m)}) = d(x_{h(g(m))}, x_{h(f(m))}) \leq \sum_{i=0}^{\nu} d(y_i, y_{i+1}) < (\nu + 1)\,\varepsilon,$$

which implies a contradiction. Therefore $\{x_n\}$ is Cauchy. □

Proposition 2.4. *Let (X, d) be a compact ν-generalized metric space and let $\{x_n\}$ be a sequence in X and let z be an element of X. Assume that there is no subsequence of $\{x_n\}$ converging to any $v \in X \setminus \{z\}$. Then $\{x_n\}$ converges exclusively to z.*

Proof. Let $\{x_{f(n)}\}$ be a subsequence in $\{x_n\}$. Since X is compact, there exists a convergent subsequence $\{x_{f(g(n))}\}$ of $\{x_{f(n)}\}$. From the assumption, $\{x_{f(g(n))}\}$ converges to z. Therefore by Proposition 2.3 (i), $\{x_n\}$ itself converges to z. From the assumption again, $\{x_n\}$ converges exclusively to z. □

The following are connected with the continuity of d.

Proposition 2.5. *Let (X, d) be a 2-generalized metric space. Let $\{x_n\}$ and $\{y_n\}$ be sequences in X converging to u and v, respectively. Assume that for each $n \in \mathbb{N}$, four elements of x_n, y_n, u, v are all different. Then*

$$d(u, v) = \lim_{n\to\infty} d(x_n, y_n) \tag{2.1}$$

holds.

Remark 2.6. See also the proof of (iii) of Example 4.1 and the remark below the proof.

Proof. By (N3), we have

$$\limsup_{n\to\infty} d(x_n, y_n) \leq \lim_{n\to\infty} \Big(d(x_n, u) + d(u, v) + d(v, y_n) \Big) = d(u, v).$$

By (N3) again, we have

$$d(u, v) \leq \liminf_{n\to\infty} \Big(d(u, x_n) + d(x_n, y_n) + d(y_n, v) \Big) = \liminf_{n\to\infty} d(x_n, y_n).$$

Therefore we obtain the desired result. □

Proposition 2.7. *Let (X, d) be a ν-generalized metric space. Let $\{x_n\}$ and $\{y_n\}$ be sequences in X converging to u and v in the strong sense, respectively. Then* (2.1) *holds.*

Remark 2.8. See also the remark below the proof of Example 4.1.

Proof. In the case where $\nu = 1$, the conclusion is obvious. So we assume $\nu \geq 2$. We first note that

$$\{n \in \mathbb{N} : \ x_n = a\}$$

is a finite set for any $a \in X \setminus \{u\}$. We also note that $\{n \in \mathbb{N} : y_n = a\}$ is a finite set for any $a \in X \setminus \{v\}$. We consider the following four cases:

- $x_n = u$ and $y_n = v$ for sufficiently large $n \in \mathbb{N}$;
- $x_n \neq u$ for infinitely many n; and $y_n = v$ for sufficiently large $n \in \mathbb{N}$;
- $x_n = u$ for sufficiently large $n \in \mathbb{N}$; and $y_n \neq v$ for infinitely many n;
- $x_n \neq u$ for infinitely many n; and $y_n \neq v$ for infinitely many n.

In the first case, (2.1) obviously holds. Let us prove (2.1) in the fourth case. Fix $\varepsilon > 0$. Then there exists $\ell \in \mathbb{N}$ such that

$$d(x_n, u) < \varepsilon, \quad \sup_{m>n} d(x_n, x_m) < \varepsilon \quad \text{and} \quad d(y_n, v) < \varepsilon$$

for any $n \geq \ell$. Fix $n \in \mathbb{N}$ with $n \geq \ell$. Since $\{x_n : n \in \mathbb{N}\}$ is an infinite set, we can choose $k_1, k_2, \ldots, k_{\nu-1}$ such that $k_i > n$,

$$\{u, v, x_n, y_n\} \cap \{x_{k_1}, x_{k_2}, \ldots, x_{k_{\nu-1}}\} = \varnothing$$

and $x_{k_1}, x_{k_2}, \ldots, x_{k_{\nu-1}}$ are all different. Further, we consider the following cases:

(i) $x_n = u$ and $y_n = v$;
(ii) $u = v$ and $x_n = y_n$;
(iii) $u = v$, $x_n \neq u$ and $y_n = v$;
(iv) $u = v$, $x_n \neq y_n$, $x_n \neq u$ and $y_n \neq v$;
(v) $u \neq v$, $x_n = y_n$, $x_n \neq u$ and $y_n \neq v$;
(vi) $u \neq v$, $x_n \neq y_n$, $x_n \neq u$ and $y_n = v$;
(vii) $u \neq v$, $x_n \neq y_n$, $x_n \neq u$, $y_n \neq v$ and $\nu = 2$;
(viii) $u \neq v$, $x_n \neq y_n$, $x_n \neq u$, $y_n \neq v$ and $\nu \geq 3$.

We note that we do not have to consider the following cases because in the fourth case, the conditions on $\{x_n\}$ and $\{y_n\}$ are the same.

(ix) $u = v$, $x_n = u$ and $y_n \neq v$;
(x) $u \neq v$, $x_n \neq y_n$, $x_n = u$ and $y_n \neq v$.

We also note that in the case where $u \neq v$, we may assume that $x_n \neq v$ and $y_n \neq u$. In the cases of (i) and (ii), $d(x_n, y_n) = d(u, v)$ holds, which implies

$$\big|d(x_n, y_n) - d(u, v)\big| < (\nu + 1)\,\varepsilon. \tag{2.2}$$

In the case of (iii), $d(u, v) = 0$ holds by (N1). We have

$$d(x_n, y_n) = d(x_n, u) < \varepsilon,$$

which implies (2.2). In the case of (iv), we have

$$\begin{aligned} d(x_n, y_n) &\leq d(x_n, x_{k_1}) + \sum_{i=1}^{\nu-2} d(x_{k_i}, x_{k_{i+1}}) + d(x_{k_{\nu-1}}, u) + d(v, y_n) \\ &< (\nu + 1)\,\varepsilon, \end{aligned}$$

which implies (2.2). In the case (v), $d(x_n, y_n) = 0$ holds. We have

$$\begin{aligned} d(u, v) &\leq d(u, x_{k_1}) + \sum_{i=1}^{\nu-2} d(x_{k_i}, x_{k_{i+1}}) + d(x_{k_{\nu-1}}, x_n) + d(y_n, v) \\ &< (\nu + 1)\,\varepsilon, \end{aligned}$$

which implies (2.2). In the case of (vi), we have

$$\begin{aligned} d(x_n, y_n) &\leq d(x_n, x_{k_1}) + \sum_{i=1}^{\nu-2} d(x_{k_i}, x_{k_{i+1}}) + d(x_{k_{\nu-1}}, u) + d(u, v) \\ &< \nu\,\varepsilon + d(u, v) \end{aligned}$$

and

$$\begin{aligned} d(u, v) &\leq d(u, x_{k_1}) + \sum_{i=1}^{\nu-2} d(x_{k_i}, x_{k_{i+1}}) + d(x_{k_{\nu-1}}, x_n) + d(x_n, y_n) \\ &< \nu\,\varepsilon + d(x_n, y_n), \end{aligned}$$

which imply (2.2). In the case of (vii), we have

$$d(x_n, y_n) \leq d(x_n, u) + d(u, v) + d(v, y_n) < 2\,\varepsilon + d(u, v)$$

and

$$d(u, v) \leq d(u, x_n) + d(x_n, y_n) + d(y_n, v) < 2\,\varepsilon + d(x_n, y_n),$$

which imply (2.2). In the case of (viii), we have

$$\begin{aligned} d(x_n, y_n) &\leq d(x_n, x_{k_1}) + \sum_{i=1}^{\nu-3} d(x_{k_i}, x_{k_{i+1}}) + d(x_{k_{\nu-2}}, u) + d(u, v) + d(v, y_n) \\ &< \nu\,\varepsilon + d(u, v) \end{aligned}$$

and

$$\begin{aligned} d(u, v) &\leq d(u, x_{k_1}) + \sum_{i=1}^{\nu-3} d(x_{k_i}, x_{k_{i+1}}) + d(x_{k_{\nu-2}}, x_n) + d(x_n, y_n) + d(y_n, v) \\ &< \nu\,\varepsilon + d(x_n, y_n), \end{aligned}$$

which imply (2.2). We have shown (2.2) in all cases. Hence (2.1) holds in the fourth case. Let us prove (2.1) in the second case. Without loss of generality, we may assume $y_n = v$. So we note that we do not have to consider the cases of (ix) nor (x). Therefore we can prove (2.1) as in the proof of the fourth case. Similarly we can prove (2.1) in the third case. □

3. Main results

We prove generalizations of Edelstein's fixed point theorem.

Theorem 3.1. *Let (X, d) be a compact 2-generalized metric space. Let T be a mapping on X such that*

$$d(Tx, Ty) < d(x, y) \tag{3.1}$$

for any $x, y \in X$ with $x \neq y$. Then T has a unique fixed point z. Moreover $\{T^n x\}$ converges exclusively to z for any $x \in X$.

Remark 3.2. We do not know whether Theorem 3.1 holds for ν-generalized metric spaces with $\nu \geq 3$.

Before proving Theorem 3.1, we need the following lemma.

Lemma 3.3. *Let (X, d) be a 2-generalized metric space and let T be a mapping on X such that*

$$d(Tx, Ty) \le d(x, y) \tag{3.2}$$

for any $x, y \in X$. Let $\{x_n\}$ be a sequence in X converging to some $v \in X$. Then $\{Tx_n\}$ converges to Tv. Moreover, if $Tx_n \ne x_n$ and $Tx_n \ne Tv$ for sufficiently large $n \in \mathbb{N}$ and $Tv \ne v$, then

$$d(v, Tv) = \lim_{n\to\infty} d(x_n, Tx_n) \tag{3.3}$$

holds.

Proof. By (3.2), we have

$$\lim_{n\to\infty} d(Tx_n, Tv) \le \lim_{n\to\infty} d(x_n, v) = 0$$

and hence $\{Tx_n\}$ converges to Tv. We assume that $Tx_n \ne x_n$ and $Tx_n \ne Tv$ for sufficiently large $n \in \mathbb{N}$ and $Tv \ne v$. Then we note $x_n \ne v$ for sufficiently large $n \in \mathbb{N}$. Since $\{x_n\}$ and $\{Tx_n\}$ converge to v and Tv respectively and $v \ne Tv$, we also note $Tx_n \ne v$ and $x_n \ne Tv$ for sufficiently large $n \in \mathbb{N}$. Therefore x_n, Tx_n, v, Tv are all different for sufficiently large $n \in \mathbb{N}$. So by Proposition 2.5, we obtain (3.3). □

Proof of Theorem 3.1. We note (3.2) holds for any $x, y \in X$. Put

$$\beta = \inf \{ d(x, Tx) : \ x \in X \}.$$

We shall show that T has a fixed point, dividing the following three cases:

- $\beta = 0$ and $d(z, Tz) = \beta$ for some $z \in X$;
- $\beta > 0$ and $d(v, Tv) = \beta$ for some $v \in X$;
- $d(x, Tx) > \beta$ for any $x \in X$.

In the first case, such z is a fixed point. In the second case, we have by (3.1)

$$\beta \le d(Tv, T^2 v) < d(v, Tv) = \beta.$$

This is a contradiction, thus, the second case cannot be possible. In the third case, we note $Tx \ne x$ for any $x \in X$. We choose a sequence $\{x_n\}$ in X such that $\{d(x_n, Tx_n)\}$ is strictly decreasing and converges to β. We note that x_n are all different. Since X is compact, there exists a subsequence $\{f(n)\}$ of $\{n\}$ such that $\{x_{f(n)}\}$ converges to some $v \in X$. We note that $\{Tx_{f(n)}\}$ converges to Tv by Lemma 3.3. In the case where $Tx_{f(n)} = Tv$ for sufficiently large $n \in \mathbb{N}$, we have by (3.2)

$$\beta < d(Tv, T^2 v) = \lim_{n\to\infty} d(Tx_{f(n)}, T^2 x_{f(n)}) \le \lim_{n\to\infty} d(x_{f(n)}, Tx_{f(n)}) = \beta,$$

which implies a contradiction. In the other case, we can choose a subsequence $\{f(g(n))\}$ of $\{f(n)\}$ such that $Tx_{f(g(n))} \ne Tv$ for $n \in \mathbb{N}$. Then by Lemma 3.3, we have

$$\beta < d(v, Tv) = \lim_{n\to\infty} d(x_{f(g(n))}, Tx_{f(g(n))}) = \beta,$$

which also implies a contradiction. Therefore the third case cannot be possible. We have shown that T has a fixed point. Let $y \in X \setminus \{z\}$ be a fixed point. Then we have by (3.1)

$$d(y, z) = d(Ty, Tz) < d(y, z),$$

which implies a contradiction. Therefore the fixed point z is unique. Fix $x \in X$. Arguing by contradiction, we assume that there exists a subsequence $\{T^{f(n)}x\}$ of $\{T^n x\}$ converging to some $v \in X \setminus \{z\}$. Then $T^n x \neq z$ holds for any $n \in \mathbb{N}$. So $\{d(T^n x, T^{n+1}x)\}$ is strictly decreasing and hence it converges to some γ and $T^n x$ are all different. Since X is compact, there exists a subsequence $\{T^{f(g(n))-1}x\}$ of $\{T^{f(n)-1}x\}$ converging to some $u \in X$. By Lemma 3.3, we have

$$Tu = T \lim_{n\to\infty} T^{f(g(n))-1}x = \lim_{n\to\infty} T^{f(g(n))}x = \lim_{n\to\infty} T^{f(n)}x = v.$$

By Lemma 3.3 again, we have

$$\lim_{n\to\infty} T^{f(g(n))+1}x = Tv = T^2 u.$$

From $v \neq z$, we have $u \neq Tu \neq T^2 u$. By Lemma 3.3, we have

$$d(Tu, T^2 u) = d(u, Tu) = \gamma.$$

By (3.1), we obtain $\gamma = 0$, which contradicts $u \neq Tu$. Therefore by Proposition 2.4, $\{T^n x\}$ converges exclusively to z. □

Theorem 3.4. *Let (X, d) be a ν-generalized metric space such that X is compact in the strong sense. Let T be a mapping on X such that (3.1) holds for any $x, y \in X$ with $x \neq y$. Then T has a unique fixed point z. Moreover for any $x \in X$, $\{T^n x\}$ converges to z in the strong sense.*

Proof. We note (3.2) holds for any $x, y \in X$. Fix $x \in X$. Then since X is compact in the strong sense, there exists a subsequence $\{T^{f(n)}x\}$ of a sequence $\{T^n x\}$ converging to some $z \in X$ in the strong sense. By (3.2), we have $\{T^{f(n)+1}x\}$ and $\{T^{f(n)+2}x\}$ converge to Tz and $T^2 z$ in the strong sense, respectively. Noting $\{d(T^n x, T^{n+1}x)\}$ is nonincreasing, we have

$$d(z, Tz) = d(Tz, T^2 z) = \lim_{n\to\infty} d(T^n x, T^{n+1}x)$$

by Proposition 2.7. Since $d(z, Tz) > 0$ contradicts (3.1), we obtain z is a fixed point of T. We can prove the uniqueness of fixed point as in the proof of Theorem 3.1. □

4. Counterexample

We finally give an example which d is not continuous.

Example 4.1 ([6])**.** Let

$$X = \big\{(0, 0)\big\} \cup \big((0, 1] \times [0, 1]\big).$$

Define a function d from $X \times X$ into $[0, \infty)$ by

$$\begin{aligned}
&d(x, x) = 0; \\
&d\big((0, 0), (s, 0)\big) = d\big((s, 0), (0, 0)\big) = s && \text{if } s \in (0, 1]; \\
&d\big((s, 0), (p, q)\big) = d\big((p, q), (s, 0)\big) = |s - p| + q && \text{if } s, p, q \in (0, 1];
\end{aligned}$$

$$d(x,y) = 3 \qquad \text{otherwise.}$$

Then the following hold:

(i) (X, d) is a 2-generalized metric space and does not have a topology which is compatible with d.
(ii) For each pair of distinct points $u, v \in X$ there is a number $r_{u,v} > 0$ such that for every $w \in X$, $r_{u,v} \leq d(u,w) + d(w,v)$.
(iii) d is not continuous.

Remark 4.2. See Proposition 2 in [5].

Proof. (i) is proved in [6]. (ii) is obvious. In order to prove (iii), we define sequences $\{u_n\}$ and $\{v_n\}$ in X by $u_n = (1/2, 1/2^n)$ and $v_n = (1/2, 1/3^n)$. Then both $\{u_n\}$ and $\{v_n\}$ converge to $(1/2, 0)$. However, we have

$$d\big((1/2,0),(1/2,0)\big) = 0 \neq 3 = \lim_{n\to\infty} d(u_n, v_n).$$

Therefore d is not continuous. □

Remark 4.3. Connected with Propositions 2.5 and 2.7, we also give another example on (iii). Define sequences $\{u_n\}$ and $\{v_n\}$ in X by $u_n = (1/3, 0)$ and $v_n = (2/3, 1/2^n)$. Then $\{u_n\}$ and $\{v_n\}$ converge to $(1/3, 0)$ and $(2/3, 0)$, respectively. We note that $\{u_n\}$ is Cauchy. However,

$$d\big((1/3,0),(2/3,0)\big) = 3 \neq 1/3 = \lim_{n\to\infty} d(u_n, v_n)$$

holds.

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