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# SOFT CONTRACTION THEOREM

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Abstract. In this paper, the concept of a soft contraction mapping on a soft metric space is introduced and the study of fixed points of such mappings is initiated.

## 1. Introduction

Mathematical models have been used extensively in real world problems related to engineering, computer sciences, economics, social, natural and medical sciences etc. It has become very common to use mathematical tools to study the behavior and different aspects of a system and its different subsystems. So it is very natural to deal with uncertainties and imprecise data in various situations. Fuzzy set theory has been evolved in mathematics as an important tool (initiated by Zadeh [36]) to resolve the issues of uncertainty and ambiguity. The contribution made by probability theory, fuzzy set theory, vague sets, rough sets and interval mathematics to deal with uncertainty is of vital importance. But there are certain limitations and deficiencies pertaining to the parametrization in fuzzy set theory (see [24]). The problem of inadequacy of parameters has been successfully solved by soft set theory which aims to provide enough tools to deal with uncertainty in a data and to represent it in a useful way. The distinguishing attribute of soft set theory is that unlike probability theory and fuzzy set theory, it does not uphold a precise quantity. This attribute has facilitated applications in decision making, demand analysis, forecasting, information sciences, mathematics and other disciplines (see for detailed survey [6, 7, 9, 11, 12, 18, 25, 27, 37, 39]).

A lot of activity has been shown in soft set theory (see e.g. [1, 2, 10, 13–17, 19, 20, 22, 23, 26, 30–34, 38]) since Molodtsov ( [24]) initiated the concept of a soft set.

The notion of soft topology on a soft set was introduced by Cagman et. al ([3]) and some basic properties of soft topological spaces were studied.

Das and Samanta introduced in [5] the notion of soft real sets, soft real numbers and discussed their properties. They also gave applications of these concepts in real life problems. Based on these notions, they introduced in [4] the concept of a soft metric. They showed that soft metric space is also a soft topological space. Recently, Wardowski ([29]) introduced a notion of soft mapping and obtained its fixed point in the setup of soft topological spaces. Motivated by this recent work, the purpose of this paper is to initiate the study of fixed point in soft metric spaces. We introduce the concept of soft contraction mapping on soft metric spaces and

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obtain a soft contraction theorem. We believe that this will open some new avenues of research in soft fixed point theory.

## 2. Preliminaries

In this section, we begin with some basic definitions and concepts related to soft sets and soft metrics needed in the sequel. This section is mainly based on the definitions and results from [4,5,7,8,21] and [24]. In some cases, we have set our own notations, terminology and made some stylistic changes to the original definitions.

Let $U$ be a given universe, $E$ a set of parameters, and $\mathbb{R}$ the set of real numbers. Throughout this paper, $P(U)$ and $B(\mathbb{R})$ denotes a family of all subsets of $U$ and the collection of all nonempty bounded subsets of $\mathbb{R}$, respectively.

**Definition 2.1** ([24]). If $F$ is a set valued mapping on a nonempty subset $A$ of $E$ taking values in $P(U)$, then the pair $(F, A)$ is called a soft set over $U$. The soft set is a parametrized family of subsets of the set $U$. For each $\varepsilon$ in $A$, the set $F(\varepsilon)$ in $U$ is called $\varepsilon-$approximate element of a soft set $(F, A)$.

We denote the collection of soft sets over a common universe $U$ by $S(U)$.

**Definition 2.2** ([8]). Let $(F, A)$, $(G, B) \in S(U)$. We say that $(F, A)$ is a soft subset of $(G, B)$ or $(G, B)$ is super soft set of $(F, A)$ if $A \subseteq B$ and for all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$. We denote it as $(F, A)\tilde{\subset}(G, B)$.

**Definition 2.3** ([8]). Let $(F, A)$, $(G, B) \in S(U)$. $(F, A)$ is said to be soft equal to $(G, B)$, if $(F, A)\tilde{\subset}(G, B)$ and $(G, B)\tilde{\subset}(F, A)$.

**Definition 2.4** ([8]). The complement $(F, A)^c$ of a soft set $(F, A)$, also denoted by $(F^c, A)$, is the multivalued mapping $F^c : A \to P(U)$ defined by $F^c(\varepsilon) = U - F(\varepsilon)$, for each $\varepsilon \in A$.

**Definition 2.5** ([21]). A soft set $(F, A)$ is said to be: (a) an absolute soft set denoted by $\tilde{U}$ if for each $\varepsilon \in A$, $F(\varepsilon) = U$; (b) a null soft set denoted by $\Phi$ if for each $\varepsilon \in A$, $F(\varepsilon) = \phi$.

**Definition 2.6** ([21]). Let $(F, A), (G, B) \in S(U)$. Union of $(F, A)$ and $(G, B)$, denoted by $(F, A)\tilde{\cup}(G, B)$, is a soft set $(H, A \cup B)$ defined as follows:

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B \\ G(\varepsilon) & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon) & \text{if } \varepsilon \in A \cap B \end{cases}.$$

**Definition 2.7** ([7]). Let $(F, A), (G, B) \in S(U)$. Intersection of $(F, A)$ and $(G, B)$, denoted by $(F, A)\tilde{\cap}(G, B)$, is the soft set $(H, A \cap B)$ defined as $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$ for each $\varepsilon$ in $A$.

**Definition 2.8** ([28]). Let $(F, A), (G, B) \in S(U)$. Difference of $(F, A)$ and $(G, B)$ denoted by $(F, A)\backslash(G, B)$ is the soft set $(H, A)$ defined as $H(\varepsilon) = F(\varepsilon)\backslash G(\varepsilon)$ for each $\varepsilon$ in $A$.

**Proposition 2.9** ([28]). *Let $(F, A), (G, B) \in S(U)$. Then*

$$\begin{aligned} ((F, A)\tilde{\cup}(G, A))^c &= (F, A)^c\tilde{\cap}(G, A)^c \text{ and} \\ ((F, A)\tilde{\cap}(G, A))^c &= (F, A)^c\tilde{\cup}(G, A)^c \end{aligned}$$

**Definition 2.10** ([5]). If $f$ is a single valued mapping on $A \subset E$ taking values in $U$, then the pair $(f, A)$, or simply $f$, is called a soft element of $U$. Let $(F, A) \in S(U)$. A soft element $f$ of $U$ is said to belongs to $(F, A)$, denoted by $f\tilde{\in}(F, A)$, if $f(e) \in F(e)$, for each $e \in A$.

**Definition 2.11** ([5]). Let $A$ be a nonempty subset of $E$. A soft real set denoted by $(\widehat{f}, A)$, or simply by $\widehat{f}$, is a mapping $\widehat{f} : A \to B(\mathbb{R})$. If $\widehat{f}$ is a single valued mapping on $A \subset E$ taking values in $\mathbb{R}$, then the pair $(\widehat{f}, A)$ or simply $\widehat{f}$, is called a soft element of $\mathbb{R}$ or a soft real number. If $\widehat{f}$ is a single valued mapping on $A \subset E$ taking values in the set $\mathbb{R}^+$ of non negative real numbers, then a pair $(\widehat{f}, A)$, or simply $\widehat{f}$, is called a non negative soft real number. We shall denote the set of non negative soft real numbers by $\mathbb{R}(A)^*$. A null soft number $\overline{0}$ is a soft real number defined by $\overline{0}(e) = 0$ for all $e \in A$. A unit soft number $\overline{1}$ is a soft real number defined by $\overline{1}(e) = 1$ for all $e \in A$. A constant soft real number $\overline{c}$ is a soft real number such that for each $e \in A$, we have $\overline{c}(e) = c$, where $c$ is some real number.

**Definition 2.12** ([4]). A soft set $(F, A)$ over $U$ is said to be a soft point if there is exactly one $e \in A$ such that $F(e) = \{x\}$ for some $x \in U$ and $F(\varepsilon) = \phi$, for all $\varepsilon \in A\backslash\{e\}$. We shall denote such a soft point by $(F_e^x, A)$ or simply by $F_e^x$ .

**Definition 2.13** ([4]). Let $(F, A)$ be a soft set over $U$. A soft point $F_e^x$ is said to belong to $(F, A)$, denoted by $F_e^x\tilde{\in}(F, A)$, if $F_e^x(e) = \{x\} \subset F(e)$.

**Definition 2.14** ([4]). Two soft points $F_{e_1}^x, F_{e_2}^y$ are said to be equal if $e_1 = e_2$ and $F_{e_1}^x(e_1) = F_{e_2}^y(e_2)$, i.e., $x = y$. Thus $F_{e_1}^x \neq F_{e_2}^y$ if and only if either $x \neq y$ or $e_1 \neq e_2$.

**Proposition 2.15** ([4]). *Let $(F, A)$ be a soft set over $U$. Then*

$$(F, A) = \cup\{F_e^x : F_e^x\tilde{\in}(F, A)\}$$

**Proposition 2.16** ([4]). *If $(F, A)$, $(G, A) \in S(U)$, then $(F, A)\tilde{\subset}(G, A)$ if and only if $F_e^x\tilde{\in}(F, A)$ implies that $F_e^x\tilde{\in}(G, A)$. Also, $(F, A)$ is soft equal to $(G, A)$ if and only if $F_e^x\tilde{\in}(F, A)$ if and only if $F_e^x\tilde{\in}(G, A)$.*

**Proposition 2.17** ([4]). *For a soft point $F_e^x$, the following hold:*

$$\begin{aligned} F_e^x &\tilde{\in} (F, A) \text{ if and only if } F_e^x\tilde{\notin}(F, A)^c, \\ F_e^x &\tilde{\in} (F, A)\tilde{\cup}(G, A) \text{ if and only if } F_e^x\tilde{\in}(F, A) \text{ or } F_e^x\tilde{\in}(G, A), \text{ and} \\ F_e^x &\tilde{\in} (F, A)\tilde{\cap}(G, A) \text{ if and only if } F_e^x\tilde{\in}(F, A) \text{ and } F_e^x\tilde{\in}(G, A). \end{aligned}$$

**Remark 2.18** ([4]). Let $\mathfrak{B}$ be a collection of soft points. The soft set generated by taking all the soft points of $\mathfrak{B}$ is denoted by $SS(\mathfrak{B})$. The collection of all soft points of $(F, A)$ is denoted $SP(F, A)$.

**Proposition 2.19** ([4]). *Let $\mathfrak{B}$, $\mathfrak{B}_1$ and $\mathfrak{B}_2$ be collections of soft points, and $(F, A)$, $(G, A) \in S(U)$. Then following hold:*

$$\begin{aligned} SP(SS(\mathfrak{B})) &= \mathfrak{B},\ SS(SP(F, A)) = (F, A), \\ SP((F, A)\tilde{\cup}(G, A)) &= SP((F, A)) \cup SP((G, A)), \\ SP((F, A)\tilde{\cap}(G, A)) &= SP((F, A)) \cap SP((G, A)) \\ SS(\mathfrak{B}_1 \cup \mathfrak{B}_2) &= SS(\mathfrak{B}_1)\tilde{\cup}SS(\mathfrak{B}_2) \text{ and} \end{aligned}$$

$$SS(\mathfrak{B}_1 \cap \mathfrak{B}_2) = SS(\mathfrak{B}_1)\tilde{\cap}SS(\mathfrak{B}_2).$$

**Definition 2.20** ([4])**.** For two soft real numbers $\widehat{f}, \widehat{g}$ , we say that

(i) $\widehat{f}\tilde{\leq}\widehat{g}$ if $\widehat{f}(e) \leq \widehat{g}(e)$, for all $e \in A$,
(ii) $\widehat{f}\tilde{\geq}\widehat{g}$ if $\widehat{f}(e) \geq \widehat{g}(e)$, for all $e \in A$,
(iii) $\widehat{f}\tilde{<}\widehat{g}$ if $\widehat{f}(e) < \widehat{g}(e)$, for all $e \in A$, and
(iv) $\widehat{f}\tilde{>}\widehat{g}$ if $\widehat{f}(e) > \widehat{g}(e)$, for all $e \in A$.

The definition of a soft metric introduced in [4] is given below:

**Definition 2.21.** A mapping $d : SP(\tilde{U}) \times SP(\tilde{U}) \to \mathbb{R}(A)^*$ is said to be a soft metric on $\tilde{U}$ if for any $U^x_\lambda, U^y_\mu, U^z_\gamma \tilde{\in} \tilde{U}$, the following hold

M1. $d(U^x_\lambda, U^y_\mu)\tilde{\geq}\bar{0}$,
M2. $d(U^x_\lambda, U^y_\mu) = \bar{0}$ if and only if $U^x_\lambda = U^y_\mu$.
M3. $d(U^x_\lambda, U^y_\mu) = d(U^x_\mu, U^y_\lambda)$.
M4. $d(U^x_\lambda, U^z_\gamma)\tilde{\leq}d(U^x_\lambda, U^y_\mu) + d(U^y_\mu, U^z_\gamma)$.

A soft metric space is a pair $(\tilde{U}, d)$ such that $\tilde{U}$ is a soft set and $d$ is a soft metric on $\tilde{U}$.

**Definition 2.22** ([4])**.** Let $(\tilde{U}, d)$ be a soft metric space, $\widehat{r}$ a non negative soft real number and $U^a_e \tilde{\in} \tilde{U}$. An open ball with center $U^a_e$ and radius $\widehat{r}$ is given by the set $B(U^a_e, \widehat{r}) = \{U^x_\lambda \tilde{\in} \tilde{U}; d(U^x_\lambda, U^a_e)\tilde{<}\widehat{r}\} \subset SP(\tilde{U})$. A soft set $SS(B(U^a_e, \widehat{r}))$ is called a soft open ball with center $U^a_e$ and radius $\widehat{r}$.

**Definition 2.23** ([4])**.** Let $(\tilde{U}, d)$ be a soft metric space, $\widehat{r}$ a non-negative soft real number and $U^a_e \tilde{\in} \tilde{U}$. A closed ball with center $U^a_e$ and radius $\widehat{r}$ is given by a set $B[U^a_e, \widehat{r}] = \{U^x_\lambda \tilde{\in} \tilde{U}; d(U^x_\lambda, U^a_e)\tilde{\leq}\widehat{r}\} \subset SP(\tilde{U})$. A soft set $SS(B[U^a_e, \widehat{r})])$ is called a soft closed ball with center $U^a_e$ and radius $\widehat{r}$.

**Definition 2.24** ([4])**.** Let $(F, A)$ be a soft subset in a soft metric space $(\tilde{U}, d)$. A soft point $F^a_e$ is said to be an interior point of the soft set $(F, A)$ if there exists a positive soft real number $\widehat{r}$ such that $F^a_e \in B(F^a_e, \widehat{r}) \subset SP(F, A)$.

**Definition 2.25** ([4])**.** Let $(\tilde{U}, d)$ be a soft metric space and $(F, A)$ a non-null soft subset of $\tilde{U}$. Then $(F, A)$ is soft open in $\tilde{U}$ with respect to $d$ if all soft points of $(F, A)$ are interior points of $(F, A)$.

**Definition 2.26** ([4])**.** Let $(\tilde{U}, d)$ be a soft metric space. A soft subset $(F, A)$ of $\tilde{U}$ is said to be soft closed in $\tilde{U}$ with respect to $d$ if its complement $(F, A)^c$ is soft open in $\tilde{U}$.

**Definition 2.27** ([4])**.** Let $(\tilde{U}, d)$ be a soft metric space and $(F, A)\tilde{\subset}\tilde{U}$. A soft point $U^a_e \tilde{\in} \tilde{U}$ is a soft limit point of $(F, A)$ if every soft open ball $SS(B(U^a_e, \widehat{r}))$ containing $U^a_e$ contains at least one soft point of $(F, A)$ other than $U^a_e$.

**Definition 2.28** ([4])**.** Let $(\tilde{U}, d)$ be a soft metric space and $(F, A)\tilde{\subset}\tilde{U}$. Then a soft set generated by the collection of all soft points of $(F, A)$ and soft limit points of $(F, A)$ is called soft closure of $(F, A)$ in $(\tilde{U}, A)$ and is denoted by $\overline{(F, A)}$.

**Definition 2.29** ([4]). Let $(\tilde{U}, d)$ be a soft metric space. A sequence $\{U^x_{\lambda,n}\}_n$ of soft points in $\tilde{U}$ is said to be convergent in $(\tilde{U}, d)$ if there is a soft point $U^y_\mu \tilde{\in} \tilde{U}$ such that $d(U^x_{\lambda,n}, U^y_\mu) \to \bar{0}$ as $n \to \infty$. This means for every $\widehat{\varepsilon} \tilde{>} \bar{0}$, chosen arbitrary, there exists a natural number $N = N(\tilde{\varepsilon})$ such that $d(U^x_{\lambda,n}, U^y_\mu) \tilde{<} \widehat{\varepsilon}$, whenever $n > N$.

**Proposition 2.30** ([4]). *Limit of a sequence $\{U^x_{\lambda,n}\}_n$ in a soft metric space $(\tilde{U}, d)$, if exists is unique.*

**Proposition 2.31** ([4]). *Let $(\tilde{U}, d)$ be a soft metric space and $(F, A) \tilde{\subset} \tilde{U}$. Then $U^y_\mu \tilde{\in} \tilde{U}$ is a soft limit point of $(F, A)$ if and only if there is a sequence $\{U^x_{\lambda,n}\}_n$ in $(F, A)$ other than $\{U^y_\mu\}_n$ which converges to $U^y_\mu$.*

**Proposition 2.32** ([4]). *Let $(F, A)$ be a soft subset in a soft metric space $(\tilde{U}, d)$. Then $(F, A)$ is soft closed if and only if $\{U^x_{\lambda,n}\}_n$ in $(F, A)$ which converges in $\tilde{U}$ cannot converges to a soft point of $(F, A)^c$.*

**Definition 2.33** ([4]). A sequence $\{U^x_{\lambda,n}\}_n$ of soft points in $(\tilde{U}, d)$ is said to be a Cauchy sequence in $(\tilde{U}, d)$ if corresponding to every $\widehat{\varepsilon} \tilde{>} \bar{0}$, there exists a natural number $m$ such that $d(U^x_{\lambda,i}, U^x_{\lambda,j}) \tilde{<} \widehat{\varepsilon}$, for all $i, j \geq m$. That is, $d(U^x_{\lambda,i}, U^x_{\lambda,j}) \to \bar{0}$ as $i, j \to \infty$.

**Proposition 2.34** ([4]). *Every convergent sequence $\{U^x_{\lambda,n}\}_n$ in a soft metric space $(\tilde{U}, d)$ is Cauchy and every Cauchy sequence is bounded.*

**Definition 2.35** ([4]). A soft metric space $(\tilde{U}, d)$ is called complete if every Cauchy sequence in $\tilde{U}$ converges to some soft point of $(\tilde{U}, d)$. In this case, we say that the soft metric $d$ is complete.

## 3. Fixed point theorems

In the sequel, soft real numbers will be denoted with "tildes" instead of "wide-hats", i.e., we will write $\tilde{\varepsilon}, \tilde{\delta}$, etc., instead of $\widehat{\varepsilon}, \widehat{\delta}$, etc.

Now we prove the following proposition.

**Proposition 3.1.** *A soft subset $(F, A)$ of a soft complete metric space $(\tilde{U}, d)$ is soft complete if and only if $(F, A)$ is soft closed in $(\tilde{U}, d)$.*

*Proof.* Suppose that $(F, A)$ is soft complete. By Proposition 2.31, we know that for every $F^y_\mu \tilde{\in} \overline{(F, A)}$, there is a sequence $\{F^x_{\lambda,n}\}_n$ in $(F, A)$ which converges to $F^y_\mu$. As $\{F^x_{\lambda,n}\}_n$ is a Cauchy sequence (Proposition 2.34) and $(F, A)$ is soft complete, $\{F^x_{\lambda,n}\}_n$ converges in $(F, A)$. By uniqueness of the limit (Proposition 2.30) we obtain that $F^y_\mu \tilde{\in} (F, A)$. This proves that $(F, A)$ is soft closed. Conversely, if $(F, A)$ is soft closed and $\{F^x_{\lambda,n}\}_n$ is Cauchy sequence in $(F, A)$. Then by the completeness of $(\tilde{U}, d)$ we have $F^x_{\lambda,n} \to F^y_\mu \tilde{\in} \tilde{U}$, which by Proposition 2.32, further implies that $F^y_\mu \tilde{\in} (F, A)$. Hence $(F, A)$ is soft complete. □

The following definitions are somehow similar to those given by Wardowski ([29]).

**Definition 3.2.** Let $(F,A)$, $(G,A) \in S(U)$. The soft Cartesian product of $(F,A)$ and $(G,A)$, denoted by $(F,A)\tilde{\times}(G,A)$, is defined as

$$(F,A)\tilde{\times}(G,A) = \{((p_1,p_2), F(p_1) \times G(p_2)) : p_1, p_2 \in A\}.$$

**Example 3.3.** Suppose that $U = \{h_1,h_2,h_3\}$ and $A = \{p_1,p_2,p_3\}$. Define soft sets $(F,A)$ and $(G,A)$ as follows:

$$\begin{aligned}(F,A) &= \{(p_1,\{h_1,h_2\}),(p_2,\{h_2,h_3\}),(p_3,\{h_1\})\} \text{ and}\\ (G,A) &= \{(p_1,\{h_1\}),(p_2,\{h_1,h_3\}),(p_3,\{h_1,h_2\})\}.\end{aligned}$$

Then

$$\begin{aligned}(F,A)\tilde{\times}(G,A) = \{&((p_1,p_1),\{h_1,h_2\}\times\{h_1\}),((p_1,p_2),\{h_1,h_2\}\times\{h_1,h_3\}),\\ &((p_1,p_3),\{h_1,h_2\}\times\{h_1,h_2\}),((p_2,p_1),\{h_2,h_3\}\times\{h_1\}),\\ &((p_2,p_2),\{h_2,h_3\}\times\{h_1,h_3\}),((p_2,p_3),\{h_2,h_3\}\times\{h_1,h_2\}),\\ &((p_3,p_1),\{h_1\}\times\{h_1\}),((p_3,p_2),\{h_1\}\times\{h_1,h_3\}),\\ &((p_3,p_3),\{h_1\}\times\{h_1,h_2\})\}.\end{aligned}$$

**Definition 3.4.** Let $(F,A),(G,A)$ be two soft sets over a common universe $U$. A soft relation $R$ is a soft set such that $(R, A\times A)\tilde{\subset}(F,A)\tilde{\times}(G,A)$. That is,

$$(R, A\times A) = \{((p,q), U_p \times U_q) : p,q \in A, U_p \subseteq F(p), U_q \subseteq G(q)\}.$$

We will denote $((p,q), U_p\times U_q) \in (R, A\times A)$ as $(p,U_p)R(q,U_q)$.

**Example 3.5.** Let $(F,A),(G,A)$ be as in Example 3.3. Then

$$R = \{((p_1,p_1),\{(h_1,h_1)\}),((p_2,p_1),\{(h_2,h_1)\}),((p_2,p_3),\{(h_2,h_1),((h_3,h_2))\})\}$$

So we can write $(p_1,\{h_1\})R(p_1,\{h_1\})$, $(p_2,\{h_2\})R(p_1,\{h_1\})$, $(p_2,\{h_2,h_3\})R(p_3, \{h_1,h_2\})$.

**Definition 3.6.** Let $(F,A)$ and $(G,A)$ be two soft sets. A soft relation $(T, A\times A)\tilde{\subset}(F,A)\tilde{\times}(G,A)$ is called a soft mapping from $(F,A)$ to $(G,A)$ if for each soft point $F_\lambda^x\tilde{\in}(F,A)$ there exists only one soft point $F_\mu^y$ such that $F_\lambda^x T F_\mu^y$. We will denote $F_\lambda^x T F_\mu^y$ by $T(F_\lambda^x) = F_\mu^y$. If $(T, A\times A)\tilde{\subset}(F,A)\tilde{\times}(G,A)$ is soft mapping from $(F,A)$ to $(G,A)$, then we shall write it as $T : (F,A)\tilde{\rightarrow}(G,A)$.

**Example 3.7.** Let $(F,A),(G,A)$ be as in Example 3.3. Suppose that $(T, A\times A)\tilde{\subset}(F,A)\tilde{\times}(G,A)$ is defined as:

$$\begin{aligned}T = \{&((p_1,p_3),\{(h_1,h_2)\}),((p_1,p_2),\{(h_2,h_1)\}),((p_2,p_1),\{(h_3,h_1)\}),\\ &((p_2,p_2),\{(h_2,h_1)\}),((p_3,p_2),\{(h_1,h_3)\})\}.\end{aligned}$$

Therefore we can write $T(F_{p_1}^{h_1}) = F_{p_3}^{h_2}$, $T(F_{p_1}^{h_2}) = F_{p_2}^{h_1}$, $T(F_{p_2}^{h_3}) = F_{p_1}^{h_1}$, $T(F_{p_2}^{h_2}) = F_{p_2}^{h_1}$ and $T(F_{p_3}^{h_1}) = F_{p_2}^{h_3}$.

**Definition 3.8.** Let $(F,A)$ and $(G,A)$ be two soft sets and $T : (F,A)\tilde{\rightarrow}(G,A)$ a soft mapping. The image of $(H,A)\tilde{\subset}(F,A)$ under the soft mapping $T$ is the soft set, denoted by $T((H,A))$, defined as follows

$$T((H,A)) = \tilde{\cup}\{T\{F_\lambda^x\} : F_\lambda^x \tilde{\in}(H,A)\}.$$

**Definition 3.9.** Let $(F, A)$ and $(G, A)$ be two soft sets and $T : (F, A)\tilde{\rightarrow}(G, A)$ a soft mapping. The inverse image of $(Y, A)\tilde{\subset}(G, A)$-under the soft mapping $T$ is the soft set, denoted by $T^{-1}((Y, A))$, defined as:

$$T^{-1}((Y, A)) = \tilde{\cup}\{F_\lambda^x\tilde{\in}(F, A), T\{F_\lambda^x\}\tilde{\in}(Y, A)\}.$$

**Definition 3.10.** Let $(F, A)$ be a soft set and $T : (F, A)\tilde{\rightarrow}(F, A)$ a soft mapping. A soft point $F_\lambda^x\tilde{\in}(F, A)$ is called a fixed point of $T$ if $T(F_\lambda^x) = F_\lambda^x$.

**Example 3.11.** Let $U = \{h_1, h_2, h_3\}$, $A = \{p_1, p_2\}$. Define the soft set $(F, A)$ as follows

$$(F, A) = \{(p_1, \{h_1, h_2\}), (p_2, \{h_2, h_3\})\}.$$

If $T : (F, A)\tilde{\rightarrow}(F, A)$ is defined as:

$$T(F_{p_1}^{h_1}) = F_{p_1}^{h_1},\ T(F_{p_1}^{h_2}) = F_{p_2}^{h_2}, T(F_{p_2}^{h_2}) = F_{p_2}^{h_3},\ \text{and}\ T(F_{p_2}^{h_3}) = F_{p_1}^{h_2},$$

then $F_{p_1}^{h_1}$ is the fixed point of $T$.

**Definition 3.12.** Let $(\tilde{U}_1, d_1)$ and $(\tilde{U}_2, d_2)$ be two soft metric spaces. A soft mapping $T : \tilde{U}_1\tilde{\rightarrow}\tilde{U}_2$ is said to be soft continuous at a soft point $U_\lambda^x\tilde{\in}\tilde{U}$ if for every $\tilde{\varepsilon}\tilde{>}\bar{0}$, there is a $\tilde{\delta}\tilde{>}\bar{0}$ such that $d_2(T(U_\lambda^x), T(U_\mu^y))\tilde{<}\tilde{\varepsilon}$ whenever $d_1(U_\lambda^x, U_\mu^y)\tilde{<}\tilde{\delta}$. If $T$ is soft continuous at every soft point of $\tilde{U}$, we say that $T$ is soft continuous on $\tilde{U}$.

**Proposition 3.13.** *Let $(\tilde{U}, d_1)$ and $(\check{Y}, d_2)$ be two soft metric spaces. For a soft mapping $T : \tilde{U}\tilde{\rightarrow}\check{Y}$ the following are equivalent:*

(i) *$T$ is soft continuous on $\tilde{U}$.*
(ii) *For any $U_\mu^y\tilde{\in}\tilde{U}$, if $U_{\lambda,n}^x \to U_\mu^y$ in $\tilde{U}$, then $T(U_{\lambda,n}^x) \to T(U_\mu^y)$ in $\check{Y}$.*
(iii) *If $(W, A)$ is soft closed in $\check{Y}$, then $T^{-1}(W, A)$ is soft closed in $\tilde{U}$.*
(iv) *If $(V, A)$ is soft open in $\check{Y}$, then $T^{-1}(V, A)$ is soft open in $\tilde{U}$.*

*Proof.* (i)⇒(ii): Suppose that $U_{\lambda,n}^x \to U_\mu^y$ in $\tilde{U}$. Given $\tilde{\varepsilon}\tilde{>}\bar{0}$, let $\tilde{\delta}\tilde{>}\bar{0}$ such that $T(SS(B(U_\mu^y, \tilde{\delta})))\tilde{\subset}SS(B(T(U_\mu^y), \tilde{\varepsilon}))$. Then, since $U_{\lambda,n}^x \to U_\mu^y$, we have $\{U_{\lambda,n}^x\}_n$ eventually in $SS(B(U_\mu^y, \tilde{\delta}))$. But this implies that $\{T(U_{\lambda,n}^x)\}_n$ eventually in $SS(B(T(U_\mu^y), \tilde{\varepsilon}))$. Since $\tilde{\varepsilon}$ is arbitrary, this means that $T(U_{\lambda,n}^x) \to T(U_\mu^y)$.

(ii)⇒(iii) Let $(W, A)$ be soft closed in $\check{Y}$. Given $\{U_{\lambda,n}^x\}_n$ in $T^{-1}(W, A)$ such that $U_{\lambda,n}^x \to U_\mu^y$ in $\tilde{U}$, we are to show that $U_\mu^y\tilde{\in}T^{-1}(W, A)$. But $\{U_{\lambda,n}^x\}_n$ in $T^{-1}(W, A)$ implies that $\{T(U_{\lambda,n}^x)\}_n$ is in $(W, A)$, while $U_{\lambda,n}^x \to U_\mu^y$ in $\tilde{U}$ tells us that $T(U_{\lambda,n}^x) \to T(U_\mu^y)$ in $\check{Y}$ from (ii). Thus, since $(W, A)$ is soft closed, we have that $T(U_\mu^y)\tilde{\in}(W, A)$ or $U_\mu^y\tilde{\in}T^{-1}(W, A)$.

(iii)⇔(iv) It is obvious, since $T^{-1}((V, A)^c) = (T^{-1}(V, A))^c$.

(iv)⇒(i) Given $U_\lambda^x\tilde{\in}\tilde{U}$ and $\tilde{\varepsilon}\tilde{>}\bar{0}$, the set $SS(B(T(U_\lambda^x), \tilde{\varepsilon}))$ is open in $\check{Y}$ and thus, by (iv), the set $T^{-1}(SS(B(T(U_\lambda^x), \tilde{\varepsilon})))$ is open in $\tilde{U}$.

Then $SS(B(U_\lambda^x, \tilde{\delta})) \tilde{\subset} T^{-1}(SS(B(T(U_\lambda^x), \tilde{\varepsilon})))$ for some $\tilde{\delta}\tilde{>}\bar{0}$, because $U_\lambda^x\tilde{\in}$ $SS(B(T(U_\lambda^x), \tilde{\varepsilon}))$. □

**Definition 3.14.** Let $(\tilde{U}, d)$ be a soft metric space and $T : \tilde{U} \tilde{\to} \tilde{U}$ a soft mapping. Then $T$ is said to be a soft contraction if

$$d(T(U_\lambda^x), T(U_\mu^y)) \tilde{\leq} \bar{c} d(U_\lambda^x, U_\mu^y)$$

for all $U_\lambda^x$, $U_\mu^y \in U(\tilde{U})$, where $\bar{0} \leq \bar{c} < \bar{1}$. We will call $\bar{c}$ as soft contraction constant.

**Remark 3.15.** A soft contraction on a soft metric space is a soft continuous mapping.

Our main result is the following.

**Theorem 3.16.** *Let $(\tilde{U}, d)$ be a soft complete metric space, where $d : SP(\tilde{U}) \times SP(\tilde{U}) \to \mathbb{R}(A)^*$ with $A$ a (non-empty) finite set, and let $T$ be a soft contraction with soft contraction constant $\bar{c}$. Then $T$ has unique soft fixed point $U_\mu^y$. Moreover, for any soft point $U_\lambda^x$, the sequence of iterates $\{T^n(U_\lambda^x)\}_n$ converges to $U_\mu^y$, and the following hold:*

$$\begin{aligned} d(U_{\lambda,n}^x, U_{\lambda,m}^x) &\tilde{\leq} \frac{\bar{c}^m}{1-\bar{c}} d(T(U_{\lambda,0}^x), U_{\lambda,0}^x), \text{ whenever } n > m, \\ d(U_{\lambda,m}^x, U_\mu^y) &\tilde{\leq} \bar{c} d(U_{\lambda,m-1}^x, U_\mu^y), \end{aligned}$$

*and*

$$d(U_{\lambda,m}^x, U_\mu^y) \tilde{\leq} \frac{\bar{c}}{1-\bar{c}} d(U_{\lambda,m-1}^x, U_{\lambda,m}^x),$$

*where $U_{\lambda,0}^x = U_\lambda^x$ and $U_{\lambda,n+1}^x = T(U_{\lambda,n}^x)$ for all $n = 0, 1, 2 \ldots$*

*Proof.* Choose a soft point $U_\lambda^x$. Put $U_{\lambda,0}^x = U_\lambda^x$ and note that

$$\begin{aligned} d(U_{\lambda,n+1}^x, U_{\lambda,n}^x) &= d(T(U_{\lambda,n}^x), T(U_{\lambda,n-1}^x)) \tilde{\leq} \bar{c} d(U_{\lambda,n}^x, U_{\lambda,n-1}^x) \\ &\tilde{\leq} \bar{c}^2 d(U_{\lambda,n-1}^x, U_{\lambda,n-2}^x) \tilde{\leq} \cdots \tilde{\leq} \bar{c}^n d(U_{\lambda,1}^x, U_{\lambda,0}^x). \end{aligned}$$

For $n > m$, we have

$$\begin{aligned} d(U_{\lambda,n}^x, U_{\lambda,m}^x) &\tilde{\leq} d(U_{\lambda,n}^x, U_{\lambda,n-1}^x) + d(U_{\lambda,n-1}^x, U_{\lambda,n-2}^x) + \cdots + d(U_{\lambda,m+1}^x, U_{\lambda,m}^x) \\ &\tilde{\leq} (\bar{c}^{n-1} + \bar{c}^{n-2} + \cdots + \bar{c}^m) d(U_{\lambda,1}^x, U_{\lambda,0}^x) \\ &\tilde{\leq} \frac{\bar{c}^m}{1-\bar{c}} d(U_{\lambda,1}^x, U_{\lambda,0}^x). \end{aligned} \tag{3.1}$$

Now we show that $\{U_{\lambda,n}^x\}_n$ is a Cauchy sequence. Indeed, choose an arbitrary soft real number $\tilde{\varepsilon} \tilde{>} \bar{0}$. Since $A$ is finite, we can write $A = \{\lambda_1, \ldots, \lambda_k\}$. Then, for each $i \in \{1, \ldots, k\}$, there exists an $N_i \in \mathbb{N}$ such that

$$\left(\frac{\bar{c}^{N_i}}{1-\bar{c}} d(U_{\lambda,1}^x, U_{\lambda,0}^x)\right)(\lambda_i) < \tilde{\varepsilon}(\lambda_i).$$

Take $N = \max\{N_1, \ldots, N_k\}$. Therefore, for any $n > m \geq N$ and any $i \in \{1, \ldots, k\}$, we have

$$\begin{aligned} d(U_{\lambda,n}^x, U_{\lambda,m}^x)(\lambda_i) &\leq \Big(\frac{\bar{c}^m}{1-\bar{c}} d(U_{\lambda,1}^x, U_{\lambda,0}^x)\Big)(\lambda_i) \\ &\leq \Big(\frac{\bar{c}^N}{1-\bar{c}} d(U_{\lambda,1}^x, U_{\lambda,0}^x)\Big)(\lambda_i) < \tilde{\varepsilon}(\lambda_i), \end{aligned}$$

i.e.,

$$d(U^x_{\lambda,n}, U^x_{\lambda,m})\tilde{<}\tilde{\varepsilon},$$

whenever $n > m \geq N$. Hence $\{U^x_{\lambda,n}\}_n$ is a Cauchy sequence. By the completeness of $(\tilde{U}, d)$ there is a $U^y_\mu \tilde{\in} \tilde{U}$ such that $d(U^x_{\lambda,n}, U^y_\mu) \to \bar{0}$ as $n \to \infty$.

Since

$$\begin{aligned} d(U^y_\mu, T(U^y_\mu)) &\tilde{\leq} d(U^x_{\lambda,n}, U^y_\mu) + d(U^x_{\lambda,n}, T(U^y_\mu)) \\ &\tilde{\leq} d(U^x_{\lambda,n}, U^y_\mu) + \bar{c}d(U^x_{\lambda,n-1}, U^y_\mu), \end{aligned}$$

we can make the second term smaller than any $\tilde{\varepsilon}\tilde{>}\bar{0}$ as $U^x_{\lambda,n} \to U^y_\mu$. Hence $d(U^y_\mu, T(U^y_\mu)) = \bar{0}$. This implies $T(U^y_\mu) = U^y_\mu$. So $U^y_\mu$ is a fixed point of $T$.

Now if $U^z_\gamma$ is another fixed point of $T$, then

$$d(U^y_\mu, U^z_\gamma) = d(T(U^y_\mu), T(U^z_\gamma))\tilde{\leq}\bar{c}d(U^y_\mu, U^z_\gamma)$$

implies that $d(U^y_\mu, U^z_\gamma) = \bar{0}$ as $\bar{c}\tilde{<}\bar{1}$. Hence $U^y_\mu = U^z_\gamma$. Therefore the fixed point of $T$ is unique.

As for $n > m$, we have

$$\begin{aligned} d(U^x_{\lambda,n}, U^x_{\lambda,m}) &\tilde{\leq} \frac{\bar{c}^m}{1-\bar{c}} d(U^x_{\lambda,1}, U^x_{\lambda,0}) \\ &= \frac{\bar{c}^m}{1-\bar{c}} d(T(U^x_{\lambda,0}), U^x_{\lambda,0}). \end{aligned}$$

Taking limit as $n \to \infty$, we obtain that

$$\begin{aligned} d(U^x_{\lambda,m}, U^y_\mu) &= d(T(U^x_{\lambda,m-1}), T(U^y_\mu)) \\ &\tilde{\leq} \bar{c}d(U^x_{\lambda,m-1}, U^y_\mu) \\ &\tilde{\leq} \bar{c}[d(U^x_{\lambda,m-1}, U^x_{\lambda,m}) + d(U^x_{\lambda,m}, U^y_\mu)]. \end{aligned}$$

This implies

$$d(U^x_{\lambda,m}, U^y_\mu)\tilde{\leq}\frac{\bar{c}}{1-\bar{c}} d(U^x_{\lambda,m-1}, U^x_{\lambda,m})$$

□

**Remark 3.17.** When $T : \tilde{U}\tilde{\to}\tilde{U}$ is a soft contraction with constant $\bar{c}$, any iterate $T^n$ is a soft contraction with constant $\bar{c}^n$. The unique soft fixed point of $T$ will also be the unique soft fixed point of $T^n$.

**Corollary 3.18.** *Let $(\tilde{U}, d)$ be a soft complete metric space, where $d : SP(\tilde{U}) \times SP(\tilde{U}) \to \mathbb{R}(A)^*$ with $A$ a (non-empty) finite set, $T : \tilde{U}\tilde{\to}\tilde{U}$ a soft contraction and $Y\tilde{\subset}\tilde{U}$ a soft closed subset such that $T(Y)\tilde{\subset}Y$. Then the unique soft fixed point of $T$ is a soft point of $Y$.*

*Proof.* Since $Y$ is a soft closed subset of a soft complete metric space, it is soft complete. Then by applying soft contraction mapping theorem to $T$ on $Y$, we obtain a soft fixed point of $T$ in $Y$. Since $T$ has only one fixed point in $\tilde{U}$, it must lie in $Y$. □

**Theorem 3.19.** *Let $T$ be a soft mapping on a soft complete metric space $(\tilde{U}, d)$, where $d : SP(\tilde{U}) \times SP(\tilde{U}) \to \mathbb{R}(A)^*$ with $A$ a (non-empty) finite set . Suppose $T$ is a soft contraction on a soft closed ball $SS(B[U^x_{\lambda,0}, \tilde{r}])$ with soft contraction constant $\bar{c}$ and $d(T(U^x_{\lambda,0}), U^x_{\lambda,0}) \tilde{<} (1-\bar{c})\tilde{r}$. Then $T$ has a unique soft fixed point in $SS(B[U^x_{\lambda,0}, \tilde{r}])$.*

*Proof.* Construct a sequence $\{U^x_{\lambda,m}\}_m$ as in the previous theorem starting from $U^x_{\lambda,0}$. Now taking $m = 0$ in (3.1) and changing $n$ to $m$, we have

$$\begin{aligned} d(U^x_{\lambda,m}, U^x_{\lambda,0}) &\tilde{\leq} \frac{1}{1-\bar{c}} d(U^x_{\lambda,1}, U^x_{\lambda,0}) \\ &= \frac{1}{1-\bar{c}} d(T(U^x_{\lambda,0}), U^x_{\lambda,0}) \tilde{<} \tilde{r}. \end{aligned}$$

Hence all $U^x_{\lambda,m}$'s are in $SS(B[U^x_{\lambda,0}, \tilde{r}])$. Since $\{U^x_{\lambda,m}\}_m$ is a Cauchy sequence, by the completeness of $(\tilde{U}, d)$ we have $U^x_{\lambda,m} \to U^y_\mu \tilde{\in} \tilde{U}$. As $SS(B[U^x_{\lambda,0}, \tilde{r}])$ is soft closed, so $U^y_\mu \tilde{\in} SS(B[U^x_{\lambda,0}, \tilde{r}])$. Hence the result. □

We conclude the paper with two examples that illustrate our main result. They are based on the following interesting example given in [4, Example 4.3].

**Example 3.20.** Let $U$ and $A$ be non-empty subsets of $\mathbb{R}$. Define $d : SP(\tilde{U}) \times SP(\tilde{U}) \to \mathbb{R}(A)^*$ as

$$d(U^x_\lambda, U^y_\mu) = |\overline{x} - \overline{y}| + \left|\overline{\lambda} - \overline{\mu}\right|,$$

for all $U^x_\lambda, U^y_\mu \in \tilde{U}$, where $|.|$ denotes the modulus of soft real numbers (recall that for each $x \in U$, $\overline{x}$ is the constant soft real number defined by $\overline{x}(\lambda) = x$ for all $\lambda \in A$, and, similarly, for each $\lambda \in A$, $\overline{\lambda}$ is the constant soft real number defined by $\overline{\lambda}(\mu) = \lambda$ for all $\mu \in A$). Then, according to [4, Example 4.3], $d$ is a soft metric on $\tilde{U}$.

**Example 3.21.** Let $U = [0, 1]$ and $E = A = \{0, 1\}$. Let $d$ be the soft metric on $\widetilde{U}$ as constructed in Example 3.20. Since $U$ is complete for the Euclidean metric, it immediately follows that $(\tilde{U}, d)$ is complete.

Now define a soft mapping $T : \tilde{U} \to \tilde{U}$ as $T(U^x_0) = U^0_0$ and $T(U^x_1) = U^{x/2}_0$ for all $x \in U$. We show that $T$ is a soft contraction with soft contraction constant $\overline{c}$ given by $\overline{c}(0) = \overline{c}(1) = 1/2$. Indeed, for each $x, y \in U$ we have $d(T(U^x_0), T(U^y_0)) = \overline{0}$, and

$$d(T(U^x_1), T(U^y_1)) = d(U^{x/2}_0, U^{y/2}_0) = \left|\overline{x/2} - \overline{y/2}\right| = \overline{c}\, |\overline{x} - \overline{y}| .$$

Finally, since $x, y \in [0, 1]$, one has for $\mu = 0, 1$,

$$\left|\overline{y/2} - \overline{0}\right|(\mu) = y/2 \leq (|x-y| + 1)/2 = (\overline{c}(|\overline{x} - \overline{y}| + \left|\overline{1} - \overline{0}\right|))(\mu),$$

so

$$d(T(U^x_0), T(U^y_1)) = d(U^0_0, U^{y/2}_0) = \left|\overline{y/2} - \overline{0}\right| \tilde{\leq} \overline{c}(|\overline{x} - \overline{y}| + \left|\overline{1} - \overline{0}\right|) = \overline{c} d(U^x_0, U^y_1).$$

Consequently, all conditions of Theorem 3.16 are satisfied. In fact, $U^0_0$ is the unique fixed point of $T$.

Our last example shows that condition "$A$ is a finite set" cannot be omitted in Theorem 3.16.

**Example 3.22.** Let $U = A = \{1/n : n \in \mathbb{N}\}$. Let $d$ be the soft metric on $\widetilde{U}$ as constructed in Example 3.20. We show that $(\widetilde{U}, d)$ is complete. Indeed, suppose that $\{U^x_{\lambda,n}\}_n$ is a Cauchy sequence in $(\widetilde{U}, d)$. Take the soft real number $\widetilde{\varepsilon}$ such that $\widetilde{\varepsilon}(\lambda) = \lambda$ for all $\lambda \in A$, i.e., $\widetilde{\varepsilon}(1/k) = 1/k$ for all $k \in \mathbb{N}$. Then, there is $m \in \mathbb{N}$ such that

$$d(U^x_{\lambda,i}, U^x_{\lambda,j}) \widetilde{<} \widetilde{\varepsilon},$$

for all $i, j \geq m$. This implies that $d(U^x_{\lambda,i}, U^x_{\lambda,j})(1/k) < \widetilde{\varepsilon}(1/k)$ for all $k \in \mathbb{N}$. Hence

$$(|\overline{x_i} - \overline{x_j}| + |\overline{\lambda_i} - \overline{\lambda_j}|)\Big(\frac{1}{k}\Big) < \frac{1}{k},$$

for all $i, j \geq m$ and for all $k \in \mathbb{N}$. Consequently

$$|x_i - x_j| + |\lambda_i - \lambda_j| < \frac{1}{k},$$

for all $i, j \geq m$ and for all $k \in \mathbb{N}$. In particular, for any $j \geq m$,

$$|x_j - x_{j+1}| + |\lambda_j - \lambda_{j+1}| < \frac{1}{k},$$

for all $k \in \mathbb{N}$.

Therefore $x_j = x_{j+1}$ and $\lambda_j = \lambda_{j+1}$ for all $j \geq m$. We deduce that $x_j = x_m$ and $\lambda_j = \lambda_m$ for all $j \geq m$. Thus the sequence $\{U^x_{\lambda,n}\}_n$ is eventually constant, and hence convergent. We conclude that $(\widetilde{U}, d)$ is complete.

Now let $T : \widetilde{U} \to \widetilde{U}$ defined as $T(U^x_\lambda) = U^{x/2}_1$ for all $x \in U$, $\lambda \in A$. Clearly $T$ has no fixed point. However it is a soft contraction with soft constant contraction $\overline{c}$ defined as $\overline{c}(\lambda) = 1/2$ for all $\lambda \in A$. Indeed, fix $x, y \in U$ and $\lambda, \mu \in A$, then for each $\eta \in A$ we have

$$\begin{aligned} d(T(U^x_\lambda), T(U^y_\mu))(\eta) &= d(U^{x/2}_1, U^{y/2}_1)(\eta) = \Big|\frac{x}{2} - \frac{y}{2}\Big| \\ &\leq \frac{1}{2}(|x-y| + |\lambda - \mu|) = \overline{c}(d((U^x_\lambda), (U^y_\mu)))(\eta). \end{aligned}$$

Hence $d((U^x_\lambda), (U^y_\mu)) \widetilde{\leq} \overline{c} d(T(U^x_\lambda), T(U^y_\mu))$.

**Remark 3.23.** After writing our paper, we have discovered that Murat I. Yazar, Cigdem Gunduz (Aras) and Sadi Bayramov have established in Theorem 4.8 of their paper "Fixed point theorems of soft contractive mappings" ([35]), a similar result to our main theorem (Theorem 3.16) but without assuming that the set $A$ is finite. Our Example 3.22 shows that their result is not correct (the error seems occur on line -3 of page 9). Furthermore, if in Example 3.22 we put $x/3$ instead of $x/2$, a counterexample to Theorem 4.9 of Yazar-Gunduz-Bayramov's paper, is also obtained.

## 4. Conclusion

In this paper we put forward the notion of soft contraction mappings based on the theory of soft element of soft metric space. We study fixed points of fuzzy soft contraction mappings and obtain, among others results, a theorem of Banach contraction principle type. Employing these results, we can further study fixed point theory in the framework of soft metric spaces.

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