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CONTRACTIVITY AND GENERICITY RESULTS FOR A CLASS OF NONLINEAR MAPPINGS

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Dedicated to the memory of Professor Francesco Saverio De Blasi

Abstract. For a class of generalized nonexpansive self-mappings of a bounded, closed and convex subset of a Banach space, we introduce the notion of a contractive mapping. We show that every contractive mapping has a unique fixed point which uniformly attracts all the iterates of the mapping. We also show that in the sense of Baire category, most mappings in this class are contractive.

1. Introduction and main results

Let $(X, \|\cdot\|)$ be a Banach space and let K be a bounded, closed and convex subset of X. Let $f : X \to [0, \infty)$ be a continuous function such that $f(0) = 0$, the set $f(K - K)$ is bounded, and the following three properties hold:

(P1) for each $\epsilon > 0$, there exists $\delta > 0$ such that if $x, y \in K$ satisfy $f(x - y) \leq \delta$, then $\|x - y\| \leq \epsilon$;

(P2) for each $\lambda \in (0, 1)$, there is $\phi(\lambda) \in (0, 1)$ such that

$$f(\lambda(x - y)) \leq \phi(\lambda) f(x - y) \text{ for all } x, y \in K;$$

(P3) the function $(x, y) \mapsto f(x - y)$, $x, y \in K$, is uniformly continuous on $K \times K$.

Denote by $\mathcal{A}$ the set of all continuous mappings $A : K \to K$ such that

$$f(Ax - Ay) \leq f(x - y) \text{ for all } x, y \in K. \tag{1.1}$$

Set

$$\operatorname{diam}(K) := \sup\{\|x - y\| : \ x, y \in K\}. \tag{1.2}$$

For each $A : K \to K$, let A^0 denote the identity operator.

For each $A, B \in \mathcal{A}$, set

$$d(A, B) := \sup\{\|Ax - Bx\| : \ x \in K\}. \tag{1.3}$$

It is clear that $(\mathcal{A}, d)$ is a complete metric space.

This class of mappings was introduced in [2], where the authors studied the existence of fixed points of these mappings by using the Baire category approach, which we now recall.

Let M be a complete metric space. According to Baire's theorem, the intersection of every countable collection of open and everywhere dense subsets of M is everywhere dense in M. This rather simple, yet powerful result has found many

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applications. In particular, given a property which elements of the space M may have, it is of interest to determine whether this property is *generic*, that is, whether the set of elements which do enjoy this property contains a countable intersection of open and everywhere dense sets. Such an approach, when a certain property is investigated for the whole space M and not just for a single point in M, has already been successfully applied in many areas of Analysis. See, for example, [1–7] and the references therein.

Using this approach, we have recently [2] established a generic fixed point theorem which we now state.

Theorem 1.1. *There exists a set $\mathcal{F}$ which is a countable intersection of open and everywhere dense subsets of $\mathcal{A}$ such that:*

1. *Each $C \in \mathcal{F}$ has a unique fixed point $x_C \in K$, that is, a unique point satisfying $Cx_C = x_C$.*
2. *For each $C \in \mathcal{F}$ and each $\epsilon > 0$, there exist a neighborhood $\mathcal{U}$ of C in $\mathcal{A}$ and a natural number n_ϵ such that for each $B \in \mathcal{U}$ and each integer $n \geq n_\epsilon$,*

$$\|B^n x - x_C\| \leq \epsilon$$

for all $x \in K$.

Note that the classical result of De Blasi and Myjak [1] is a particular case of our result where $f = \|\cdot\|$. As a matter of fact, the mappings studied here can be considered generalized nonexpansive mappings with respect to f. Such an approach, where in some problems of functional analysis the norm is replaced by a general function, was used in [4, 5, 6] in the study of generalized best approximation problems, which we now recall.

Given a closed subset S of a Banach space X and a point $x \in X$, we consider in [4, 5, 6] the minimization problem

$$(P) \qquad \min\{f(x-y) : y \in S\}.$$

This problem was studied by many mathematicians mostly in the case where $f(x) = \|x\|$. In this special case it is well known that if S is convex and X is reflexive, then problem (P) always has at least one solution. This solution is unique when X is strictly convex. In [4] and [5] we establish the generic solvability and well-posedness of problem (P) for a general function f.

Set

$$D_f := \sup\{f(x-y) : x, y \in K\}. \tag{1.4}$$

A mapping $A \in \mathcal{A}$ is called (f)-*contractive* if there exists a decreasing function $\psi : [0, \infty) \to [0, 1]$ such that

$$\psi(t) < 1 \text{ for all } t > 0, \tag{1.5}$$

and

$$f(Ax - Ay) \leq \psi(f(x-y))f(x-y) \text{ for all } x, y \in K. \tag{1.6}$$

In the case where $f(x) = \|x\|$, our definition coincides with the classical definition of a contractive mapping used in the literature [6, Section 1.3]. In this case it is known that a contractive mapping has a unique fixed point which attracts uniformly all the iterates of the mapping (see [6, Section 3.1] and the references mentioned there). In the present paper we extend this result to the general case. We also show that a generic (typical) mapping belonging to the space $\mathcal{A}$ is (f)-contractive. In the case where $f(x) = \|x\|$, this result can be found in [6, Section 3.1] and in the references mentioned there.

More precisely, in this paper we establish the following results.

Theorem 1.2. *Let a mapping $A \in \mathcal{A}$ be (f)-contractive. Then there exists a unique fixed point $\bar{x} \in K$ satisfying $A\bar{x} = \bar{x}$.*

Theorem 1.2 is proved in Section 2.

Theorem 1.3. *Let a mapping $A \in \mathcal{A}$ be (f)-contractive, assume that a point $\bar{x} \in K$ satisfies*

$$A\bar{x} = \bar{x}$$

and let $\epsilon > 0$. Then there exist $\delta > 0$ and a natural number $n_0 > 2$ such that for each integer $n \geq n_0$ and each sequence $\{x_i\}_{i=1}^n \subset K$ which satisfies

$$\|x_{i+1} - Ax_i\| \leq \delta$$

for all $i \in \{1, \dots, n-1\}$, the following inequality holds:

$$\|x_i - \bar{x}\| \leq \epsilon, \ i = n_0, \dots, n.$$

Corollary 1.4. *Let a mapping $A \in \mathcal{A}$ be (f)-contractive and assume that $\bar{x} \in K$ satisfies*

$$A\bar{x} = \bar{x}.$$

Then $A^i x \to \bar{x}$ as $i \to \infty$ for all $x \in K$, uniformly on K.

Theorem 1.3 is our stable convergence result. It follows from property (P1) and the next result, which is proved in Section 3.

Theorem 1.5. *Let a mapping $A \in \mathcal{A}$ be (f)-contractive, assume that $\bar{x} \in K$ satisfies*

$$A\bar{x} = \bar{x}$$

and let $\epsilon > 0$. Then there exist $\delta > 0$ and a natural number $n_0 > 2$ such that for each integer $n \geq n_0$ and each sequence $\{x_i\}_{i=1}^n \subset K$ which satisfies

$$\|x_{i+1} - Ax_i\| \leq \delta$$

for all $i \in \{1, \dots, n-1\}$, the following inequality holds:

$$f(x_i - \bar{x}) \leq \epsilon, \ i = n_0, \dots, n.$$

The following generic result is proved in Section 4.

Theorem 1.6. *There exists a set $\mathcal{F}$ which contains a countable intersection of open and everywhere dense subsets of $\mathcal{A}$ such that each element in $\mathcal{F}$ is an (f)-contractive mapping.*

2. Proof of Theorem 1.2

By assumption, there exists a decreasing function $\psi : [0, \infty) \to [0, 1]$ such that (1.5) and (1.6) hold.

Let $\epsilon > 0$ be given. We claim that there exists a point $x_\epsilon \in K$ such that

$$f(x_\epsilon - Ax_\epsilon) \leq \epsilon.$$

Fix $x \in K$ and consider the sequence $\{A^i x\}_{i=0}^{\infty}$. We assert that there exists an integer $i \geq 0$ such that

$$f(A^i x - A^{i+1}x) \leq \epsilon.$$

Assume the contrary. Then for each integer $i \geq 0$,

$$f(A^i x - A^{i+1}x) > \epsilon. \tag{2.1}$$

Let $i \geq 0$ be an integer. Then (2.1) holds. Since the function ψ is decreasing, it follows from (1.5), (1.6) and (2.1) that

$$\begin{aligned} f(A^{i+1}x - A^{i+2}x) &\leq f(A(A^i x) - A(A^{i+1}x)) \\ &\leq \psi(f(A^i x - A^{i+1}x))f(A^i x - A^{i+1}x) \\ &\leq \psi(\epsilon)f(A^i x - A^{i+1}x) \end{aligned}$$

and so,

$$\begin{aligned} f(A^i x - A^{i+1}x) - f(A^{i+1}x - A^{i+2}x) &\geq (1 - \psi(\epsilon))f(A^i x - A^{i+1}x) \\ &\geq (1 - \psi(\epsilon))\epsilon. \end{aligned} \tag{2.2}$$

By (2.2), for any natural number n,

$$\begin{aligned} f(x - Ax) &\geq f(A^0 x - A^1 x) - f(A^n x - A^{n+1}x) \\ &= \sum_{i=0}^{n-1}[f(A^i x - A^{i+1}x) - f(A^{i+1}x - A^{i+2}x)] \\ &\geq (1 - \psi(\epsilon))\epsilon n \to \infty \text{ as } n \to \infty. \end{aligned}$$

The contradiction we have reached proves that there indeed exists an integer $i \geq 0$ such that

$$f(A^i x - A^{i+1}x) \leq \epsilon,$$

as asserted. Thus we have shown that the following property holds:

(P4) for each $\epsilon > 0$, there exists a point $x_\epsilon \in K$ such that

$$f(x_\epsilon - Ax_\epsilon) \leq \epsilon.$$

We now show that the following property also holds:

(P5) for each $\epsilon > 0$, there exists $\delta > 0$ such that for each $x, y \in K$ satisfying

$$f(x - Ax) \leq \delta \text{ and } f(y - Ay) \leq \delta,$$

the inequality $\|x - y\| \leq \epsilon$ holds.

To this end, let $\epsilon > 0$ be given. By (P1), there exists $\epsilon_1 \in (0, \epsilon)$ such that

$$\text{if } x, y \in K \text{ and } f(x - y) \leq \epsilon_1, \text{ then } \|x - y\| \leq \epsilon. \tag{2.3}$$

Since the function $(x, y) \mapsto f(x-y)$, $x, y \in K$, is uniformly continuous on $K \times K$, there exists $\epsilon_2 \in (0, \epsilon_1)$ such that for each $z_1, z_2, z_3, z_4 \in K$ satisfying

$$\|z_1 - z_3\|,\ \|z_2 - z_4\| \leq \epsilon_2,$$

the following inequality holds:

$$|f(z_1 - z_2) - f(z_3 - z_4)| \leq (1 - \psi(\epsilon_1))\epsilon_1/8. \tag{2.4}$$

By property (P1), there exists $\delta \in (0, \epsilon_2)$ such that

$$\text{if } z_1, z_2 \in K \text{ satisfy } f(z_1 - z_2) \leq \delta, \text{ then } \|z_1 - z_2\| \leq \epsilon_2. \tag{2.5}$$

Let $x, y \in K$ satisfy

$$f(x - Ax) \leq \delta \ \text{ and } \ f(y - Ay) \leq \delta. \tag{2.6}$$

We claim that the inequality $\|x - y\| \leq \epsilon$ holds.

Assume the contrary. Then

$$\|x - y\| > \epsilon. \tag{2.7}$$

By (2.3) and (2.7),

$$f(x - y) > \epsilon_1. \tag{2.8}$$

Since the function ψ is decreasing, it follows from (1.6) and (2.8) that

$$f(Ax - Ay) \leq \psi(f(x - y))f(x - y) \leq \psi(\epsilon_1)f(x - y)$$

and

$$f(x - y) - f(Ax - Ay) \geq (1 - \psi(\epsilon_1))f(x - y) \geq (1 - \psi(\epsilon_1))\epsilon_1. \tag{2.9}$$

In view of (2.5) and (2.6),

$$\|x - Ax\|,\ \|y - Ay\| \leq \epsilon_2. \tag{2.10}$$

By (2.10) and the choice of ϵ_2 (see (2.4)),

$$|f(x - y) - f(Ax - Ay)| \leq (1 - \psi(\epsilon_1))\epsilon_1/8.$$

This contradicts (2.9). The contradiction we have reached proves that, in fact, $\|x - y\| \leq \epsilon$. Thus property (P5) holds.

By property (P4), there exists a sequence $\{x_i\}_{i=1}^{\infty} \subset K$ such that

$$\lim_{i \to \infty} f(x_i - Ax_i) = 0. \tag{2.11}$$

In view of (2.11) and property (P5), $\{x_i\}_{i=1}^{\infty}$ is a Cauchy sequence. Therefore there exists

$$\bar{x} = \lim_{i \to \infty} x_i \tag{2.12}$$

in the norm topology of X. By (2.11), (2.12) and the continuity of A and f,

$$f(\bar{x} - A\bar{x}) = 0$$

and in view of property (P1), it follows that

$$A\bar{x} = \bar{x}.$$

Assume that $y \in K$ satisfies $Ay = y$. If $y \neq \bar{x}$, then by (1.5) and (1.6),

$$f(\bar{x} - y) = f(A\bar{x} - Ay) \leq f(\bar{x} - y)\psi(f(\bar{x} - y)) < f(\bar{x} - y),$$

a contradiction. The contradiction we have reached completes the proof of Theorem 1.2.

3. Proof of Theorem 1.5

By assumtion, there exists a decreasing function $\psi : [0, \infty) \to [0, 1]$ such that (1.5) and (1.6) hold.

By property (P3), there exists a number $\delta \in (0, \epsilon)$ such that

$$|f(y_1 - y_2) - f(z_1 - z_2)| \leq (1 - \psi(\epsilon/2))\epsilon/8 \tag{3.1}$$

for each $y_1, y_2, z_1, z_2 \in K$ satisfying

$$\|y_i - z_i\| \leq \delta, \ i = 1, 2.$$

Choose a natural number

$$n_0 > 2 + 2\epsilon^{-1}(1 - \psi(\epsilon))^{-1} \sup\{f(z_1 - z_2) : \ z_1, z_2 \in K\}. \tag{3.2}$$

Assume that $n \geq n_0$ is an integer and that a sequence $\{x_i\}_{i=1}^n \subset K$ satisfies

$$\|x_{i+1} - Ax_i\| \leq \delta \tag{3.3}$$

for all $i \in \{1, \dots, n - 1\}$.

We claim that there exists an integer $j \in \{1, \dots . n_0\}$ such that

$$f(x_j - \bar{x}) \leq \epsilon.$$

Indeed, assume the contrary. Then for all $i \in \{1, \dots, n_0\}$,

$$f(x_i - \bar{x}) > \epsilon. \tag{3.4}$$

Let $i \in \{1, \dots, n_0\}$. Then (3.4) holds. By (1.6), (3.4) and the equality $A\bar{x} = \bar{x}$,

$$\begin{aligned} f(Ax_i - \bar{x}) &\leq \psi(f(x_i - \bar{x}))f(x_i - \bar{x}) \\ &\leq \psi(\epsilon)f(x_i - \bar{x}) \end{aligned}$$

and

$$f(x_i - \bar{x}) - f(Ax_i - \bar{x}) \geq (1 - \psi(\epsilon))f(x_i - \bar{x}) \geq (1 - \psi(\epsilon))\epsilon. \tag{3.5}$$

By (3.3) and the choice of δ (see (3.1)),

$$|f(Ax_i - \bar{x}) - f(x_{i+1} - \bar{x})| \leq (1 - \psi(\epsilon/2))\epsilon/8.$$

When combined with (3.5), this implies that

$$f(x_i - \bar{x}) - f(x_{i+1} - \bar{x}) \geq (1 - \psi(\epsilon))\epsilon/2. \tag{3.6}$$

In view of (3.6),

$$\begin{aligned} f(x_1 - \bar{x}) &\geq f(x_1 - \bar{x}) - f(x_{n_0+1} - \bar{x}) \\ &= \sum_{i=1}^{n_0} [f(x_i - \bar{x}) - f(x_{i+1} - \bar{x})] \\ &\geq n_0(1 - \psi(\epsilon))\epsilon/2 \end{aligned}$$

and

$$n_0 \leq 2\epsilon^{-1}(1 - \psi(\epsilon))^{-1} f(x_1 - \bar{x}).$$

This contradicts (3.2). The contradiction we have reached proves that there exists $j \in \{1, \dots, n_0\}$ such that

$$f(x_j - \bar{x}) \leq \epsilon,$$

as claimed. Assume now that an integer $i \in \{j, \dots, n\} \setminus \{n\}$ satisfies

$$f(x_i - \bar{x}) \leq \epsilon. \tag{3.7}$$

There are two cases:

$$f(x_i - \bar{x}) \leq \epsilon/2; \tag{3.8}$$

$$f(x_i - \bar{x}) > \epsilon/2 \tag{3.9}$$

Assume that (3.8) holds. By (1.1), (3.8) and the equality $A\bar{x} = \bar{x}$,

$$f(Ax_i - \bar{x}) \leq \epsilon/2. \tag{3.10}$$

In view of (3.3) and the choice of δ (see (3.1)),

$$|f(x_{i+1} - \bar{x}) - f(Ax_i - \bar{x})| \leq \epsilon/8.$$

When combined with (3.10), this implies that

$$f(x_{i+1} - \bar{x}) \leq f(Ax_i - \bar{x}) + |f(x_{i+1} - \bar{x}) - f(Ax_i - \bar{x})| \leq \epsilon.$$

Assume that (3.9) holds. By (1.6), (3.7) and (3.9),

$$\begin{aligned} f(Ax_i - \bar{x}) &\leq \psi(f(x_i - \bar{x}))f(x_i - \bar{x}) \\ &\leq \psi(\epsilon/2)f(x_i - \bar{x}) \\ &\leq \psi(\epsilon/2)\epsilon. \end{aligned} \tag{3.11}$$

In view of (3.3) and the choice of δ (see (3.1)),

$$|f(x_{i+1} - \bar{x}) - f(Ax_i - \bar{x})| \leq (1 - \psi(\epsilon/2))\epsilon/8.$$

When combined with (3.11), this implies that

$$\begin{aligned} f(x_{i+1} - \bar{x}) &\leq f(Ax_i - \bar{x}) + |f(x_{i+1} - \bar{x}) - f(Ax_i - \bar{x})| \\ &\leq \psi(\epsilon/2))\epsilon + (1 - \psi(\epsilon/2))\epsilon/8 < \epsilon. \end{aligned}$$

Thus in both cases

$$f(x_{i+1} - \bar{x}) \leq \epsilon. \tag{3.12}$$

Therefore

$$f(x_j - \bar{x}) \leq \epsilon$$

and if an integer $i \in \{j, \dots, n\} \setminus \{n\}$ satisfies (3.7), then (3.12) also holds. This implies that (3.7) holds for all $i = j, \dots, n$. Theorem 1.5 is proved.

4. Proof of Theorem 1.6

We use the convention that the supremum over the empty set is zero.

For each natural number n, we denote by $\mathcal{F}_n$ the set of all $A \in \mathcal{A}$ such that

$$\sup\{f(Ax - Ay)f(x - y)^{-1} : \ x, y \in K \text{ and } f(x - y) \geq n^{-1}D_f\} < 1. \tag{4.1}$$

Define

$$\mathcal{F} = \cap_{n=1}^{\infty} \mathcal{F}_n. \tag{4.2}$$

Let $A \in \mathcal{F}$ be given. We now show that the mapping A is (f)-contractive. To this end, set

$$\psi(0) = 1$$

and for each $t > 0$, put

$$\psi(t) = \sup\{f(Ax - Ay)f(x - y)^{-1} : \ x, y \in K \text{ and } f(x - y) \geq t\}. \tag{4.3}$$

Clearly, $\psi : [0, \infty) \to [0, 1]$ is a decreasing function. In view of (4.1) and (4.3),

$$\psi(t) < 1 \text{ for all } t > 0.$$

By property (P1) and (4.3), for all $x, y \in K$ satisfying $x \neq y$,

$$f(x - y) > 0$$

and

$$f(Ax - Ay)f(x - y)^{-1} \leq \psi(f(x - y)).$$

Thus the mapping A is (f)-contractive.

Let n be a natural number. In order to complete the proof of the theorem, it is sufficient to show that $\mathcal{F}_n$ contains an open and everywhere dense subset of $\mathcal{A}$.

Fix

$$\theta \in K.$$

Let $A \in \mathcal{A}$ and $r \in (0, 1]$. Fix a number $\gamma \in (0, 1)$ such that

$$\gamma \text{diam}(K) \leq r/4. \tag{4.4}$$

Set

$$A_\gamma x = (1 - \gamma)Ax + \gamma\theta, \ x \in K. \tag{4.5}$$

Clearly,

$$A_\gamma(K) \subset K.$$

By (4.4) and (4.5),

$$\begin{aligned} d(A_\gamma, A) &= \sup\{\|(1 - \gamma)Ax + \gamma\theta - Ax\| : x \in K\} \\ &\leq \sup\{\gamma\|\theta - Ax\| : x \in K\} \\ &\leq \gamma \text{diam}(K) \leq r/4. \end{aligned} \tag{4.6}$$

In view of (P3), there exists $\delta \in (0, 1)$ such that for each $y_1, y_2, z_1, z_2 \in K$ satisfying

$$\|y_i - z_i\| \leq \delta, \ i = 1, 2,$$

we have

$$|f(y_1 - y_2) - f(z_1 - z_2)| \leq (1 - \phi(1 - \gamma))(8n)^{-1}D_f. \tag{4.7}$$

Assume that

$$B \in \mathcal{A} \text{ and } d(B, A_\gamma) \leq \delta. \tag{4.8}$$

By (1.1), (4.5) and property (P3), for each $x, y \in K$,

$$\begin{aligned} f(A_\gamma x - A_\gamma y) &= f((1-\gamma)Ax - (1-\gamma)Ay)) \\ &= f((1-\gamma)(Ax - Ay)) \\ &\leq \phi(1-\gamma) f(x-y). \end{aligned} \tag{4.9}$$

Let $x, y \in K$ satisfy

$$f(x-y) \geq n^{-1} D_f. \tag{4.10}$$

It follows from (1.3), (4.8) and the choice of δ (see (4.7)) that

$$|f(Bx - By) - f(A_\gamma x - A_\gamma y)| \leq (1 - \phi(1-\gamma))(8n)^{-1} D_f. \tag{4.11}$$

In view of (4.9), (4.10) and (4.11),

$$\begin{aligned} f(Bx - By) &\leq f(A_\gamma x - A_\gamma y) + |f(Bx - By) - f(A_\gamma x - A_\gamma y)| \\ &\leq \phi(1-\gamma) f(x-y) + 8^{-1}(1 - \phi(1-\gamma)) n^{-1} D_f \\ &\leq \phi(1-\gamma) f(x-y) + 8^{-1}(1 - \phi(1-\gamma)) f(x-y) \end{aligned}$$

and

$$\begin{aligned} f(Bx - By) f(x-y)^{-1} &\leq \phi(1-\gamma) + 8^{-1}(1 - \phi(1-\gamma)) \\ &= (7/8)\phi(1-\gamma) + 1/8 < 1. \end{aligned}$$

Since the above relation holds for all $x, y \in K$ satisfying (4.10), we conclude that $B \in \mathcal{F}_n$. Thus each mapping B satisfying (4.8) belongs to $\mathcal{F}_n$. In view of (4.6), we see that $\mathcal{F}_n$ contains an open and everywhere dense subset of $\mathcal{A}$. This completes the proof of Theorem 1.6.

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