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GENERALIZED DARBO'S THEOREM AND ITS APPLICATION

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Dedicated to Prof. Wataru Takahashi on his 70th birthday

Abstract. In the present paper, using a mapping $H : \mathbb{R}^+ \to \mathbb{R}$, we introduce a new type of set contraction called H-set contraction and prove a new fixed point theorem concerning H-set contraction which is a generalization of Darbo's fixed point theorem. For the some concrete mappings H, we obtain the set contractions of the type known from the literature and also k-set contraction. The article includes the examples of H-set contractions and some examples showing that the obtained extension is significant. We also show the applicability of the obtained results to the theory of functional integral equations.

1. Introduction

Probably one of the most important fixed point theorems in nonlinear analysis is the Schauder fixed point theorem which states that a compact operator T which maps a nonempty, closed, convex, bounded subset M of a Banach space into itself, has a fixed point in M. In 1955, Darbo took an essential step in extending the Schauder fixed point Theorem using the idea of a measure of noncompactness, defined in 1939 by Kuratowski. He introduced a new class of mappings, the so-called $k-$set contractions, using the Kuratowski's measure of noncompactness. Darbo's theorem is not only of theoretical interest, but also has found a great number of applications in both linear and nonlinear analysis. Typically, such applications are characterized by some "loss of compactness" which arises in many fields such as boundary value problems, imbedding theorems, Schrödinger operators, essential spectra, integral transforms, substitution operators over complex domains, superposition operators in function spaces, differential equations in Banach spaces, Fredholm operators, Banach space geometry, nonlinear spectral theory, bifurcation theory and functional integral equation. Due to the most important role of measures of non-compactness and suitable kinds of operators associated with them, many authors have focused on set-contractive operators and obtained a lot of valuable results (see [1, 2, 4, 5, 6, 8, 16]). In 2003 Banas [8] by using the technique of a fixed-point theorem of Darbo type, obtained an existence result for some functional-integral equation. The idea of a measure of noncompactness in proving a generalization of the Darbo fixed point theorem and its application to integral equation for mappings which are called $\varphi-$set contractions is used by Aghajani et al. [1] in 2013.

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In this paper, using a mapping $H : \mathbb{R}^+ \to \mathbb{R}$ we introduce a new type of set contraction, called $H-$set contraction and for these concrete mappings H, we obtain different kinds of set contractions of the type known from the literature, also k-set contraction(s). We investigate the conditions under which one can guarantee the existence of fixed points of this class of operators. In the other words, motivated by [1, 8], we obtain a generalization of Darbo's fixed point theorem and by using this theorem we present the existence of solutions for some nonlinear functional integral equations which include many key integral and functional equations that appear in nonlinear analysis and its applications.

A scrutiny of the proof of Darbo's theorem shows that we did not use the special definition of the Kuratowski measure of noncompactness, but only its regularity, its homogeneity, and its convex closure invariance. So Darbo's theorem holds true for any measure of noncompactness γ satisfying these conditions, in particular a function $\gamma : \{B \subset X : B \textit{ is bounded}\} \to [0, \infty)$ is said to be a measure of noncompactness on a Banach space X, if it satisfies the following conditions:

(1) (invariance under closure and convex hull): $\gamma(\bar{co}B) = \gamma(B)$,
(2) (regularity): $\gamma(B) = 0$ if and only if B is relatively compact,
(3) (semi-homogeneity): $\gamma(\alpha B) = |\alpha|\gamma(B)$ for all $\alpha \in \mathbb{R}$.

The definition of measure of noncompactness given above is more general than that of Kuratowski or Hausdorff measure of noncompactness given respectively by

$$\alpha(B) = \inf\{r > 0 : B \text{ may be covered by finitely many sets of diameter } \leq r\},$$

$$\beta(B) = \inf\{r > 0 : \text{there exists a finite } r\text{-net for } B \text{ in } X\},$$

(see [2, 4]).

A continuous operator $T : X \to X$ is said to be:

- a countable $\gamma - k-$set contraction: [14]
  if $\gamma(T(C)) \leq k\gamma(C)$ for each countable bounded set $C \subseteq X$ and for $0 \leq k < 1$,
- $\gamma-$countably condensing if $\gamma(T(C)) < \gamma(C)$ for each countable bounded set $C \subset X$ with $\gamma(C) > 0$.
- a countable $\gamma - \varphi-$set contraction: [11]
  if $\gamma(T(C)) \leq \varphi(\gamma(C))$ for some $\varphi \in \Phi = \{\varphi : \mathbb{R}^+ \to \mathbb{R}^+, \varphi(t) < t \text{ for } t > 0,\ \varphi(0) = 0\}$ and each countable bounded set $C \subseteq X$.
  Clearly, every countable $\gamma - k-$set contraction is a countable $\gamma - \varphi-$set contraction where $\varphi(t) = kt$.

2. Main results

In this section we state our main definition which determines an important class of operators including linear bounded operators, nonexpansive operators, completely continuous operators and k-set contractive operators.

Definition 2.1. Let $H : \mathbb{R}^+ \to \mathbb{R}$ be a mapping satisfying:

$(H1)$ H is strictly increasing, i.e. $H(\alpha) < H(\beta)$ for all $\alpha, \beta \in \mathbb{R}^+$ such that $\alpha < \beta$.

$(H2)$ For each sequence $\{a_n\}_{n\in\mathbb{N}}$ of positive numbers $\lim_{n\to\infty} a_n = 0$ if and only if $\lim_{n\to\infty} H(a_n) = -\infty$.

A mapping $T : X \to X$ is said to be a countable $H-$set contraction if there exists $\tau > 0$ such that for all countable bounded sets $C \subseteq X$

$$(\gamma(T(C)) > 0) \Rightarrow \tau + H(\gamma(T(C))) \leq H(\gamma(C)). \tag{2.1}$$

Remark 2.2. From $(H1)$ and (2.1) it is easy to conclude that every $H-$set contraction is a condensing map. Also, every $\gamma-$condensing map T with $\gamma(T(C)) \neq 0$ (for every countable bounded set C) is an $H-$set contraction.

Remark 2.3. Considering different types of the mapping H in (2.2), one can obtain a variety of set contractions as follows.

Example 2.4. Let $H : \mathbb{R}^+ \to \mathbb{R}$ be given by $H(x) = \ln x$. It is clear that H satisfies $(H1)$ and $(H2)$. Each mapping $T : X \to X$ satisfying (2.1) is an $H-$set contraction such that

$$\gamma(T(C)) \leq e^{-\tau}\gamma(C), \tag{2.2}$$

for all countable set $C \subset X$ with $\gamma(T(C)) > 0$. It is clear that for a countable set $C \subset X$ if $\gamma(T(C)) = 0$ the inequality (2.2) also holds, i.e. T is a countable $k-$set contraction with $k = e^{-\tau}$.

Example 2.5. Let $H : (0, \infty) \to \mathbb{R}$ be given by $H(x) = \ln(x^2 + x)$. Obviously H satisfies $(H1)$ and $(H2)$ and for any $H-$contraction T and any countable set $C \subset X$ with $\gamma(T(C)) > 0$, the following condition holds:

$$\frac{\gamma(T(C))(\gamma(T(C)) + 1)}{\gamma(C)(\gamma(C) + 1)} \leq e^{-\tau}.$$

Example 2.6. Let $H : (0, \infty) \to \mathbb{R}$ be given by $H(x) = \ln x + x$. Obviously H satisfies $(H1)$ and $(H2)$ and for any $H-$contraction T and any countable set $C \subset X$ with $\gamma(T(C)) > 0$, the following condition holds:

$$\frac{\gamma(T(C))}{\gamma(C)} e^{\gamma(T(C))-\gamma(C)} \leq e^{-\tau}.$$

Example 2.7. Let $H : (0, \infty) \to \mathbb{R}$ be given by $H(x) = \frac{-1}{\sqrt{x}}$. Then H satisfies $(H1)$ and $(H2)$ and for any $H-$contraction T, we have

$$\gamma(T(C)) \leq \frac{1}{(1 + \tau\sqrt{\gamma(C)})^2}\gamma(C),$$

for all countable set $C \subset X$ with $\gamma(T(C)) > 0$. Here we obtained a special case of $\varphi-$set contractions.

Now we state our main theorem:

Theorem 2.8. *(Generalized Darbo's theorem) Let $C \neq \emptyset$ be a bounded, closed and convex subset of a Banach space X, γ be the measure of noncompactness on X and suppose that $T : C \to C$ is a continuous H-set contraction. Then T has a fixed point.*

Proof. We put $C_0 = C$ and define a decreasing sequence of sets $C_{n+1} = co(T(C_n))$ for $n = 0, 1, \ldots.$ Then it follows by (1) and hypothesis that

$$\gamma(C_{n+1}) = \gamma(co(T(C_n))) = \gamma(T(C_n)).$$

Without loss of generality we suppose that $\gamma(T(C_n)) \neq 0$ for $n = 0, 1, \ldots$, since T is an H-set contraction there exists $\tau > 0$ such that

$$\tau + H(\gamma(T(C_n))) \leq H(\gamma(C_n)).$$

Therefore we have

$$\tau + H(\gamma(C_{n+1})) \leq H(\gamma(C_n)),$$

hence

$$H(\gamma(C_n)) \leq H(\gamma(C_{n-1})) - \tau \leq H(\gamma(C_{n-2})) - 2\tau \leq \ldots \leq H(\gamma(C_0)) - n\tau,$$

and so

$$\lim_n H(\gamma(C_n)) = -\infty.$$

Now it follows by $(H2)$ that

$$\lim_n \gamma(C_n) = 0.$$

Therefore $C_\infty = \bigcap_{n=1}^{\infty} C_n \neq \emptyset$ is compact. Since C_∞ is also closed and convex, T has a fixed point by Schauder's fixed point theorem. □

3. An application to a functional integral equation

In this section we provide applications of the generalization of Darbo's fixed point theorem contained in Theorem 2.8 to prove the existence of solutions of a functional integral equation of Volterra type. We will work in the Banach space $BC(\mathbb{R}^+)$ consisting of all real functions defined, bounded and continuous on $\mathbb{R}^+$. The space $BC(\mathbb{R}^+)$ is furnished with the standard supremum norm i.e., the norm defined by

$$||x|| = \sup\{|x(t)| : t \geq 0\}.$$

For any nonempty bounded subset X of $BC(\mathbb{R}^+)$, $x \in X$, $T > 0$ and $\epsilon \geq 0$, let

$$\omega^T(x, \varepsilon) = \sup\{|x(t) - x(s)| : s, t \in [0, T],\ |t - s| \leq \varepsilon\}, \quad X(t) = \{x(t) : x \in X\},$$

$$\begin{aligned} \omega^T(X, \varepsilon) = &\ \sup\{\omega^T(x, \varepsilon) :\ x \in X\}, \\ \omega_0^T(X) = &\ \lim_{\varepsilon \to 0} \omega^T(X, \varepsilon), \\ \omega_0(X) = &\ \lim_{T \to \infty} \omega_0^T(X), \end{aligned} \tag{3.3}$$

$$\text{diam} X(t) = \sup\{|x(t) - y(t)| : x, y \in X\}, \tag{3.4}$$

and

$$\mu(X) = \omega_0(X) + \limsup_{t \to \infty} \text{diam} X(t). \tag{3.5}$$

Banaś has shown in [7] that the function μ is a measure of noncompactness in the space $BC(\mathbb{R}^+)$. The kernel of this measure contains nonempty and bounded sets X such that functions belonging to X are locally equicontinuous on $\mathbb{R}^+$ and "the thickness of the bundle" formed by functions from X tends to zero at infinity.

Consider the following conditions:

(A_0) The function $f : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}$ is continuous but for any nonempty bounded subset X of $BC(\mathbb{R}^+)$, the family of $\{f(t,x) : x \in X\}$ is not equi-continuous and for all $t \in \mathbb{R}^+$ The function $t \to f(t,0)$ is a member of the space $BC(\mathbb{R}^+)$. Moreover there exists $\tau > 0$ such that

$$|f(t,x) - f(t,y)| \neq 0 \Rightarrow \tau + H(|f(t,x) - f(t,y)|) \leq H(|x-y|).$$

(A_1) The function $g : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}$ is continuous and there exist continuous functions $a, b : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying

$$|g(t,s,x)| \leq a(t)b(s),$$

for all $t, s \in \mathbb{R}^+$ with $s \leq t$ and $x \in \mathbb{R}$, where

$$\lim_{t\to\infty} a(t) \int_0^t b(s)ds = 0.$$

(A_2) There exists a positive solution r_0 of the inequality

$$H^{-1}(H(r_0) - \tau) + q \leq r_0,$$

where q is the constant defined by the equality

$$q = \sup\{|f(t,0)| + a(t) \int_0^t b(s)ds : t \geq 0\}.$$

Theorem 3.1. *Let $(A_0), (A_1), (A_2)$ be satisfied, then the nonlinear integral equation*

$$x(t) = f(t,x(t)) + \int_0^t g(t,s,x(s))ds, \quad t \in \mathbb{R}^+ \tag{3.6}$$

has at least one solution in the space $BC(\mathbb{R}^+)$.

Proof. Define T on the space $BC(\mathbb{R}^+)$ by

$$(Tx)(t) = f(t,x(t)) + \int_0^t g(t,s,x(s))ds, \quad for \quad t \in \mathbb{R}^+.$$

By the imposed assumptions, Tx is continuous on $\mathbb{R}^+$, further for arbitrary fixed function $x \in BC(\mathbb{R}^+)$ we have

$$\begin{aligned}|(Tx)(t)| \leq &\ |f(t,x(t)) - f(t,0)| + |f(t,0)| + \textstyle\int_0^t |g(t,s,x(s))|ds \\ \leq &\ H^{-1}(H(|x(t)|) - \tau) + |f(t,0)| + a(t) \textstyle\int_0^t b(s)ds,\end{aligned}$$

therefore

$$||Tx|| \leq H^{-1}(H(||x(t)||) - \tau) + q,$$

where $q = \sup\{|f(t,0)| + a(t) \int_0^t b(s)ds : t \in \mathbb{R}^+\}$. Since q is finite by assumption (A_2) there exists $x_0 \in BC(\mathbb{R}^+)$ such that $||x_0|| = r_0$ and

$$||Tx_0|| \leq ||x_0||.$$

Thus T maps the space $BC(\mathbb{R}^+)$ into itself and T is a self mapping of the ball B_{r_0}. In what follows we show that T is continuous on the ball B_{r_0}. In order to do this fix an arbitrary $\varepsilon > 0$. Then for $x, y \in B_{r_0}$ such that $||x-y|| \le \varepsilon$, we have

$$\begin{aligned} |(Tx)(t)-(Ty)(t)| &< |x(t)-y(t)| + \textstyle\int_0^t |g(t,s,x(s)) - g(t,s,y(s))|ds \\ &< |x(t)-y(t)| + \textstyle\int_0^t |g(t,s,x(s))|ds + \int_0^t |g(t,s,y(s))|ds \\ &\le \varepsilon + 2c(t), \end{aligned}$$

for $t \in \mathbb{R}^+$ and $c(t) = a(t)\int_0^t b(s)ds$.
Moreover by assumption (A_1) there exists a number $L > 0$ such that

$$2a(t)\int_0^t b(s)ds \le \varepsilon,$$

for each $t \ge L$.
Thus for an arbitrary $t \ge L$ we get

$$|(Tx)(t)-(Ty)(t)| \le 2\varepsilon.$$

Since the function $g(t,s,x)$ is uniformly continuous on the set $[0,L]\times[0,L]\times[-r_0,r_0]$ one can infer that $\omega^L(g,\varepsilon) \to 0$ as $\varepsilon \to 0$, where

$$\omega^L(g,\varepsilon) = \sup\{|g(t,s,x)-g(t,s,y)| : t,s \in [0,L], x,y \in [-r_0,r_0], |x-y| \le \varepsilon\}.$$

For arbitrary fixed $t \in [0,L]$ we have

$$|(Tx)(t)-(Ty)(t)| \le \varepsilon + \int_0^L \omega^L(g,\varepsilon)ds = \varepsilon + L\omega^L(g,\varepsilon).$$

Therefore we can deduce that T is continuous on B_{r_0}.
In the sequel, let us take a set $X \subseteq B_{r_0}$, $X \neq \emptyset$. Further, we fix the numbers $L > 0, \varepsilon > 0$ and a function $x \in X$. Then, choosing $t, s \in [0,L]$ such that $s < t$ and $|t-s| \le \varepsilon$ we get by our assumptions

$$\begin{aligned} |(Tx)(t)-(Tx)(s)| &\le |f(t,x(t)) - f(s,x(s))| \\ &\quad + \Big|\int_0^t g(t,\iota,x(\iota))d\iota - \int_0^s g(s,\iota,x(\iota))d\iota\Big| \\ &\le |f(t,x(t)) - f(s,x(t))| + |f(s,x(t)) - f(s,x(s))| \\ &\quad + \Big|\int_0^t g(t,\iota,x(\iota))d\iota - \int_0^t g(s,\iota,x(\iota))d\iota\Big| \\ &\quad + \Big|\int_0^t g(s,\iota,x(\iota))d\iota - \int_0^s g(s,\iota,x(\iota))d\iota\Big| \\ &\le \omega_1^L(f,\varepsilon) + |x(t)-x(s)| + \int_0^t |g(t,\iota,x(\iota)) - g(s,\iota,x(\iota))|d\iota \\ &\quad + \int_s^t |g(s,\iota,x(\iota))|d\iota \\ &\le \omega_1^L(f,\varepsilon) + \omega^L(f,\varepsilon) \\ &\quad + \int_0^t \omega_1^L(g,\varepsilon)d\iota + a(s)\int_s^t b(\iota)d\iota \\ &\le \omega_1^L(f,\varepsilon) + \omega^L(f,\varepsilon) \end{aligned}$$

$$+L\omega_1^L(g,\varepsilon)+\varepsilon\sup\{a(s)b(t):t,s\in[0,L]\}$$

where

$$\omega_1^L(f,\varepsilon)=\sup\{|f(t,x)-f(s,x)|:t,s\in[0,L],x\in[-r_0,r_0],|t-s|\leq\varepsilon\},$$

$$\omega_1^L(g,\varepsilon)=\sup\{|g(t,\iota,x)-g(s,\iota,x)|:\iota,t,s\in[0,L],x\in[-r_0,r_0],|t-s|\leq\varepsilon\}.$$

Further $\omega_1^L(f,\varepsilon)\to 0$ and $\omega_1^L(g,\varepsilon)\to 0$ as $\varepsilon\to 0$, because of the uniform continuity of f on the set $[0,L]\times[-r_0,r_0]$ and g on the set $[0,L]\times[0,L]\times[-r_0,r_0]$. Moreover since the functions $a=a(t)$ and $b=b(t)$ are continuous on $\mathbb{R}^+$, we have that

$$\sup\{a(s)b(t):t,s\in[0,L]\}$$

is finite. Hence, from (3.5) we derive

$$\omega_0^L(TX)<\omega_0^L(X)$$

and finally

$$\omega_0(TX)\leq\omega_0(X). \tag{3.7}$$

Now, choose two arbitrary functions $x,y\in X$. Then for $t\in\mathbb{R}$ we have

$$\begin{aligned}|(Tx)(t)-(Ty)(t)| &\leq |f(t,x(t))-f(t,y(t))|\\ &\quad+\int_0^t|g(t,s,x(s))|ds+\int_0^t|g(t,s,y(s))|ds\\ &< |x(t)-y(t)|+2a(t)\int_0^t b(s)ds\\ &= |x(t)-y(t)|+2c(t).\end{aligned}$$

which yields

$$diam(TX)(t)<diamX(t)+2c(t).$$

Consequently we have

$$\limsup_{t\to\infty}diam(TX)(t)\leq\limsup_{t\to\infty}diamX(t). \tag{3.8}$$

Linking (3.7) and (3.8)

$$\omega_0(TX)+\limsup_{t\to\infty}diam(TX)(t)\leq\omega_0(X)+\limsup_{t\to\infty}diamX(t),$$

or equivalently

$$\mu(TX)\leq\mu(X),$$

where μ is the measure of noncompactness defined in the space $BC(\mathbb{R}^+)$. Since TX is not equi-continuous by assumption, then $\mu(TX)\neq 0$. Therefore there exists $\tau>0$ such that

$$\tau+H(\gamma(TX))\leq H(\gamma(X)),$$

the above inequality in conjunction with Theorem 2.8 allows us to deduce that there exists a solution $x(t)$ of equation (3.6) in the space $BC(\mathbb{R}^+)$.

□

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