Yokohama Publishers
ISSN 1349-8169 ONLINE JOURNAL

Pacific Journal of Optimization
Vol. 21, No. 3, pp. 519–531, 2025

NOVEL FILLED FUNCTION ALGORITHM WITHOUT PARAMETERS FOR UNCONSTRAINED GLOBAL OPTIMIZATION*

Guanglei Sun, Youlin Shang†, Xiaoqiang Wang and Roxin Zhang

Abstract: Many optimization problems modelled from actual problems have multiple global or local minimizers, which makes traditional optimization algorithms easily plunge into the local minimizer. The filled function algorithm is an effective method for such problems, which transforms the objective function into a filled function with special properties at the current local minimizer. By iteratively minimizing the objective and the filled function, the filled algorithm can gradually discover better local minimizers until the global minimizer is obtained. Hence, constructing new forms of filled function is a popular research direction, which can improve the performance of the filled function algorithm. A new parameter-free filled function with some advantages is constructed and the corresponding theoretical analyses are conducted in this paper. Firstly, its local minimizers are those of the objective function. Secondly, the current local minimizer of the objective function is the global maximizer of the proposed filled function. Based on the proposed filled function, a new global optimization algorithm without parameters to tune is given, which can avoid alternately solving the objective function and the filled function. The new algorithm is applied to solve several test functions with satisfactory experimental results, which show that the proposed algorithm is more efficient compared to the existing filled function algorithm.

Key words: *unconstrained global optimization, filled function, global minimizer, local minimizer*

Mathematics Subject Classification: *90C26, 90C30*

---

1 Introduction

Many practical problems, arising in various fields such as machine learning, biomedicine, engineering and economics, can be transformed into the general global optimization problem:

$$\begin{aligned} &\min f(x) \\ &\text{s.t.} x \in R^n, \end{aligned} \tag{1.1}$$

where $f(x) : R^n \to R$ represents the objective function. With the development of the information age, the objective function with complicated structure is mostly a non-convex function and has multiple local minima. For solving such global optimization problems and providing an effective solution to practical problems, many new global optimization algorithms have emerged in recent years, which mainly are divided into deterministic algorithms

*This research was supported by Henan Province Natural Science Foundation Youth Science Fund (No.252300420898), Key Technologies Research and Development Program of Henan Province (Nos.252102240125, 242102210102) and National Natural Science Foundations of China (No.12071112).

†Corresponding author

 DOI: https:// doi.org/10.61208/pjo-2025-025

and randomized algorithms. The deterministic algorithm generates a series of iterative points converging to the global optimal solution based on the mathematical properties of the objective function [6, 8, 12, 21, 22], whereas randomized algorithms with random factors adopt a trial and error mechanism based on the objective function value to find a satisfactory solution in the feasible region [1, 14, 23, 24]. However, when dealing with non-convex function, the current optimization algorithms are still prone to local minima as well as slow convergence, and so on. Designing global optimization algorithm that can efficiently solve global optimization problems has been a hotspot recently.

The filled function algorithm has been proven to be a deterministic algorithm with effectiveness and practicability for solving global optimization problems [2, 3, 20, 29, 31]. The conception of the filled function algorithm was initially proposed by Ge for solving unconstrained global optimization problems, and its solving process is completed through a two-stage minimizing cycle [4]. In stage 1, a local minimizer x_1^* of the objective function $f(x)$ is discovered by applying any local minimization algorithm starting from a given initial point. In stage 2, a filled function $P(x, x_1^*)$ of $f(x)$ at x_1^* is constructed before a local minimizer x_1' of $P(x, x_1^*)$ with $f(x_1') < f(x_1^*)$ is identified by any local minimization algorithm. If the point x_1' is obtained, then the stage 1 is re-executed with x_1' as an initial point to discover a better local minimizer x_2^* of $f(x)$ satisfying $f(x_2^*) < f(x_1^*)$. The above process repeats until minimizing the filled function fails to find a better point. Then the current local minimizer of $f(x)$ is considered as the global minimizer. Obviously, one key of the filled function algorithm is to design suitable filled functions which can enable a move from one local minimizer to another better one. This has been studied by many scholars for a long time.

To this day, a large number of efficient and valuable filled functions have been constructed and applied to solve various optimization problems [4,5,7,9–11,13,15–17,19,25–28,30,32,33], such as max-cut problems, pathological analysis, and permanent magnet linear synchronous motor. However, the existing filled functions have some shortcomings, e.g., more than one parameter to adjust with sensitivity as well as ill-property. For example, the filled functions constructed in [2, 4, 5, 21, 30] contain parameters and exponential terms, leading to the algorithm failure when the parameter is improper; the filled functions proposed in [11,20] involve two parameters, increasing the difficulty of algorithm implementation; the filled functions with one parameter given in [13, 25] are discontinuous to which the traditional gradient-based optimization methods cannot be applied.

In view of this, a new parameter-free filled function is presented as well as a novel filled function method in this paper, and the main contributions are as follows:

- The filled function is parameterless, which simplifies the filled function method process;
- The filled function and the objective function have the same local minimizer in $\Omega_2 = \{x \in \Omega | f(x) < f(x_k^*)\}$, i.e., the obtained local minimizer of the filled function is a better local minimizer of the objective function;
- The function value of the filled function in $\Omega_2 = \{x \in \Omega | f(x) < f(x_k^*)\}$ is always less than that in $\Omega_1 = \{x \in \Omega \backslash \{x_k^*\} | f(x) \geq f(x_k^*)\}$;
- A novel filled function method is constructed, which eliminates the need to further minimize the objective function once a local minimizer of the filled function is obtained and reduces the steps in the traditional filled function method;
- Theoretical analysis and numerical experiments are carried out to demonstrate the effectiveness and feasibility of the new algorithm.

The rest of this paper is constructed as follows. Section 2 introduces the fundamental knowledge and assumptions used in this paper. A new filled function without parameters is given and its theoretical analyses are conducted in Section 3 before the corresponding algorithm is designed for unconstrained global optimization problems in Section 4. Section 5 executes some numerical experiments to illustrate the performance of the new filled function algorithm, and some conclusions are given in Section 6.

2 Preliminaries

Throughout this paper, some conditions of Problem (1.1) are given to ensure the implementation of the presented method.

Assumption 2.1. $f(x)$ is continuously differentiable on R^n.

Assumption 2.2. Let Ω_l be the set containing all local minimizers of $f(x)$, whose element may be infinite, but $f_l^* = \{f(x)|x \in \Omega_l\}$ is finite.

Assumption 2.3. $\lim\limits_{\|x\| \to +\infty} f(x) = +\infty$

Assumption 2.3 indicates that there exists a bounded closed set $\Omega \subset R^n$ such that $\Omega_l \subset \Omega$. Then Problem (1.1) is equivalent to the following box-constrained optimization problem:

$$\begin{array}{l} \min f(x) \\ \text{s.t.} x \in \Omega, \end{array} \tag{2.1}$$

where $\Omega = \{x \in R^n | lb_i \leq x_i \leq ub_i, i = 1, \dots, n\}$ is a box constraint. As a consequence, the following lemmas hold.

Lemma 2.1. *There exists a constant K such that $0 \leq \max\limits_{x_1, x_2 \in \Omega} \|x_1 - x_2\|^2 \leq K < +\infty$.*

Lemma 2.2. *There exists a constant M such that $|f(x)| < M$.*

Let $x_k^* \in \Omega_l$ be a local minimizer of $f(x)$, and then the feasible domain Ω in Problem (2.1) is divided into three subspaces: $\Omega_1 = \{x \in \Omega \backslash \{x_k^*\} | f(x) \geq f(x_k^*)\}$, $\Omega_2 = \{x \in \Omega | f(x) < f(x_k^*)\}$ and $\{x_k^*\}$. The modified definition of the filled function for Problem (1.1) in the paper [30] is presented.

Definition 2.3. $P(x, x_k^*)$ is called a filled function of $f(x)$ at x_k^*, if it has the following properties:

- x_k^* is a strictly local maximizer of $P(x, x_k^*)$;
- $\nabla P(x, x_k^*) \neq 0$, $\forall x \in \Omega_1$;
- $P(x, x_k^*)$ has a local minimizer in Ω_2 if x_k^* is not a global minimizer of $f(x)$.

3 A Parameter-Free Filled Function and Its Properties

In this section, a new filled function without parameters is given as follows:

$$P(x, x_k^*) = \sinh(\frac{1}{\|x - x_k^*\|^2 + 1})\theta(f(x) - f(x_k^*)) + \min(0, f(x) - f(x_k^*))^3, \tag{3.1}$$

$$\theta(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases},$$

where $\|\cdot\|$ is the Euclidean vector norm.

The following theorems prove that $P(x, x_k^*)$ is a filled function, which satisfies the conditions in Definition 2.3.

Theorem 3.1. *Let x_k^* be a local minimizer of $f(x)$, then x_k^* is a strictly local maximizer of $P(x, x_k^*)$.*

Proof. Since x_k^* is a local minimizer of $f(x)$, there exists a neighbourhood $N(x_k^*, \delta)$ of x_k^* with $\delta > 0$ such that $f(x) \geq f(x_k^*)$, $\forall x \in N(x_k^*, \delta)$. Thus, for $\forall x \neq x_k^* \in N(x_k^*, \delta)$,

$$P(x, x_k^*) = \sinh(\frac{1}{\|x - x_k^*\|^2 + 1}) < \sinh(1) = P(x_k^*, x_k^*).$$

Thus, x_k^* is a strictly local maximizer of $P(x, x_k^*)$. □

Theorem 3.2. *Let x_k^* be a local minimizer of $f(x)$, then $\nabla P(x, x_k^*) \neq 0$ for $\forall x \in \Omega_1$.*

Proof. Due to $\forall x \in \Omega_1$ implying $f(x) \geq f(x_k^*)$ and $x \neq x_k^*$, we have

$$P(x, x_k^*) = \sinh(\frac{1}{\|x - x_k^*\|^2 + 1}),$$

$$\nabla P(x, x_k^*) = \cosh(\frac{1}{\|x - x_k^*\|^2 + 1})\frac{-2(x - x_k^*)}{(1 + \|x - x_k^*\|^2)^2} \neq 0.$$

□

Meanwhile, for $\forall x \in \Omega_1$, we can also have

$$(x - x_k^*)^T P(x, x_k^*) = \cosh(\frac{1}{\|x - x_k^*\|^2 + 1})\frac{-2\|x - x_k^*\|^2}{(1 + \|x - x_k^*\|^2)^2} < 0.$$

That is, for $\forall x \in \Omega_1$, $(x - x_k^*)$ is a descent direction of $P(x, x_k^*)$ at x.

Theorem 3.3. *Let x_k^* be a local minimizer of $f(x)$.For $x_1, x_2 \in \Omega_1$ with $\|x_2 - x_k^*\| > \|x_1 - x_k^*\|$, then*

$$P(x_2, x_k^*) < P(x_1, x_k^*).$$

Proof. Based on $x_1, x_2 \in \Omega_1$, $f(x_1) \geq f(x_k^*)$ and $f(x_2) \geq f(x_k^*)$, then

$$P(x_2, x_k^*) - P(x_1, x_k^*) = \sinh(\frac{1}{\|x_2 - x_k^*\|^2 + 1}) - \sinh(\frac{1}{\|x_1 - x_k^*\|^2 + 1}).$$

Because of $\|x_2 - x_k^*\| > \|x_1 - x_k^*\|$ implying $\frac{1}{\|x_2 - x_k^*\|^2 + 1} < \frac{1}{\|x_1 - x_k^*\|^2 + 1}$, we have

$$\sinh(\frac{1}{\|x_2 - x_k^*\|^2 + 1}) < \sinh(\frac{1}{\|x_1 - x_k^*\|^2 + 1}).$$

Thus,

$$P(x_2, x_k^*) - P(x_1, x_k^*) < 0.$$

Hence, $P(x_2, x_k^*) < P(x_1, x_k^*)$. □

Theorem 3.4. *Let x_k^* be a local minimizer of $f(x)$ and Ω_2 is not empty, then for $\forall x_1 \in \Omega_1$ and $\forall x_2 \in \Omega_2$, it holds that*

$$P(x_2, x_k^*) < P(x_1, x_k^*).$$

Proof. Since $\forall x_1 \in \Omega_1$ and $\forall x_2 \in \Omega_2$, then

$$f(x_2) < f(x_k^*) \le f(x_1),$$

then,

$$P(x_2, x_k^*) - P(x_1, x_k^*) = (f(x_2) - f(x_k^*))^3 - \sinh(\frac{1}{\|x_1 - x_k^*\|^2 + 1}) < 0.$$

Hence, $P(x_2, x_k^*) < P(x_1, x_k^*)$. □

Theorems 3.1-3.4 illustrate that the following points: (1) x_k^* is a strictly global maximizer of $P(x, x_k^*)$ in Ω; (2) $P(x, x_k^*)$ has no saddle point in Ω_1, and its function value decreases as the point $x \in \Omega_1$ moves away from x_k^*; (3) If Ω_2 is not empty, the point obtained by minimizing $P(x, x_k^*)$ must strike into Ω_2, which is due to the fact that the function value of $P(x, x_k^*)$ in Ω_2 is always smaller than that in Ω_1, thus achieving the goal of escaping the current local minimizer of $f(x)$. The following theorem will prove that $P(x, x_k^*)$ and $f(x)$ have the same local minimizer in Ω_2, which also means that the global minimizer is the same.

Theorem 3.5. *Let x_k^* be a local rather than global minimizer of $f(x)$, then the local minimizers of $f(x)$ in Ω_2 are also the local minimizers of $P(x, x_k^*)$.*

Proof. Obviously, Ω_2 is not empty. Thus, $f(x)$ has a better local minimizer x_l in Ω_2. Then there exists a neighbourhood $N(x_l, \delta)$ of x_l with $\delta > 0$, such that for $\forall x \neq x_l \in N(x_l, \delta) \cap \Omega_2$, the following inequalities hold:

$$f(x_l) < f(x) < f(x_k^*),$$

$$f(x_l) - f(x_k^*) < f(x) - f(x_k^*) < 0.$$

Hence, we have

$$P(x_l, x_k^*) = (f(x_l) - f(x_k^*))^3 < (f(x) - f(x_k^*))^3 = P(x, x_k^*).$$

Thus, x_l is also a local minimizer of $P(x, x_k^*)$. □

Theorem 3.5 implies that if Ω_2 is non-empty, $P(x, x_k^*)$ must have a local minimizer in Ω_2. Thus, we give the following theorem 3.6.

Theorem 3.6. *If Ω_2 is non-empty, then the local minimizers of $P(x, x_k^*)$ in Ω_2 are the local minimizers of $f(x)$.*

Proof. Suppose $\widetilde{x}_l \in \Omega_2$ is a local minimizer of $P(x, x_k^*)$. For $\widetilde{x}_l \in \Omega_2$ with $f(\widetilde{x}_l) < f(x_k^*)$, there exists a neighbourhood $N(\widetilde{x}_l, \delta)$ of $\widetilde{x}_l$ with $\delta > 0$, such that for $\forall x \neq \widetilde{x}_l \in N(\widetilde{x}_l, \delta) \cap \Omega_2$, we can obtain

$$P(\widetilde{x}_l, x_k^*) < P(x, x_k^*),$$

$$f(\widetilde{x}_l) < f(x_k^*),$$

$$f(x) < f(x_k^*).$$

Then

$$P(\widetilde{x}_l, x_k^*) = (f(\widetilde{x}_l) - f(x_k^*))^3 < P(x, x_k^*) = (f(x) - f(x_k^*))^3.$$

Thus $f(\widetilde{x}_l) < f(x)$, which means that $\widetilde{x}_l$ is a local minimizer of $f(x)$. Therefore, $P(x, x_k^*)$ and $f(x)$ have the same local minimizers in the region Ω_2. □

Theorems 3.5 and 3.6 illustrate that if Ω_2 is non-empty, then $P(x, x_k^*)$ and $f(x)$ have the same local as well as global minimizers in Ω_2.

4 Novel Filled Function Algorithm Without Parameters for Unconstrained Global Optimization

By the theoretical discussion in the previous section, a global optimization algorithm is described as follows:

Step 0: Set k as the iterative number and $k = 1$; let $e_1, e_2, \ldots, e_{2n}$ be the positive and negative coordinate directions; randomly select one point $x_k^0 \in \Omega$.

Step 1: Minimize the objective function $f(x)$ from x_k^0 to obtain a local minimizer x_k^*.

Step 2: Construct the filled function

$$P(x, x_k^*) = \sinh\left(\frac{1}{\left\|x - x_k^*\right\|^2 + 1}\right)\theta(f(x) - f(x_k^*)) + \min\left(0, f(x) - f(x_k^*)\right)^3,$$

$$\theta(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}.$$

Set $j = 1$.

Step 3: If $j \leq 2n$, go to step 4; otherwise take x_k^* as a global minimizer of the global optimization Problem (1.1) and terminate the algorithm.

Step 4: If $j \leq n$, $\delta = rand * (ub_j - (x_k^*)_j)$; otherwise $\delta = rand * ((x_k^*)_{j-n} - lb_{j-n})$. Go to step 5.

Step 5: Set $x = x_k^* + \delta e_j$. If $x \in \Omega$, go to Step 6; otherwise, let $j = j + 1$, go to step 3.

Step 6: Minimize $P(x, x_k^*)$ from x to achieve a local minimizer $\widetilde{x_k^*}$ and continue to step 7.

Step 7: If $f(\widetilde{x_k^*}) < f(x_k^*)$, set $x_{k+1}^* = \widetilde{x_k^*}$, $k = k + 1$ and go to step 2; otherwise, set $j = j + 1$ and go to step 3.

The motivation and description of the algorithm are given as follows:

- In step 0, n is the dimension of $f(x)$, $e_j, j = 1, 2, \ldots, n$ indicate the positive coordinate directions and $e_j = -e_{j-n}, j = n + 1, \ldots, 2n$ represent the negative coordinate directions. More search directions can be selected to increase the success of the algorithm, which requires more computational cost.

- In step 1, a local minimizer x_k^* of $f(x)$ is obtained by utilizing one local search method, then a filled function $P(x, x_k^*)$ of $f(x)$ at x_k^* is constructed in Step 2.

- In step 5, the step size $\delta > 0$ can be chosen arbitrarily by step 3 and 4 to generate one new initial point $x = x_k^* + \delta e_j$, because the value of $P(x, x_k^*)$ decreases when $x \in \Omega_1$ stays away from x_k^* and the values of $P(x, x_k^*)$ in the region Ω_2 are always smaller than those in the region Ω_1. Therefore, one local minimizer $\widetilde{x_k^*}$ obtained by minimizing $P(x, x_k^*)$ from any initial point different from x_k^* falls on Ω_2.

- In step 6, a new local minimizer $\widetilde{x_k^*}$ is obtained by utilizing the hybrid Hooke and Jeeves method [18] to minimize $P(x, x_k^*)$ from the initial point x. Then, if $f(\widetilde{x_k^*}) < f(x_k^*)$, $\widetilde{x_k^*}$ is also a local minimizer of $f(x)$, and then a new filled function is constructed at $\widetilde{x_k^*}$, otherwise it indicates that we cannot find a new local minimizer and need to re-select a new search direction to minimize $P(x, x_k^*)$.

From the above description, our algorithm does not require solving the objective function again after solving the filled function, which theoretically reduces the number of iterations by half compared to traditional filled function algorithms.

5 Numerical Experiments

In order to verify the performance of the newly constructed algorithm in this paper, it is applied to some global optimization problems presented in [19]. Meanwhile, the computational results are compared with those of the algorithm in [19], which are displayed in Tables 1-6. The symbols are adopted in these tables as follows:

k: The iterative number of local minimizer obtained by minimizing $f(x)$;

x_k^* : The kth local minimizer of $f(x)$;

$f(x_k^*)$: The function value of $f(x)$ at x_k^*;

c_k : The total number of local minimizers of $f(x)$ and $P(x, x_k^*)$ discovered in the k-th iteration;

c_f : The number of function evaluations of $f(x)$ when the algorithm terminates;

c_P : The number of function evaluations of $P(x, x_k^*)$ when the algorithm terminates;

x^* : The global minimizer of $f(x)$;

$f(x^*)$: The optimal value of $f(x)$ at x^*;

$No.$: The number of the Example;

$t(s)$: The CPU running time when the algorithm terminates.

Note: The local minimizers of our filled function are also the local minimizers of the objective function, while the filled function in the paper [19] does not possess this property. This implies that $c_k = k$ in this paper, whereas $c_k = 2k - 1$ in the paper [19].

Example 5.1. 2-dimensional function with multiple local minimizers

$$\begin{aligned}&\min f(x) = [1 - 2x_2 + 0.2\sin(4\pi x_2) - x_1]^2 + [x_2 - 0.5\sin(2\pi x_1)]^2\\&\text{s.t. } 0 \le x_1 \le 10, -10 \le x_2 \le 0.\end{aligned}$$

This problem has the global minimizer $x^* = (1, 0)$ with $f(x^*) = 0$.

Our algorithm successfully finds an approximate global minimizer x^* = (1.8974, -0.3005) with $f(x^*)$ = 3.9293e-15 through three iterations, which is significantly better than the results in the paper [19] with regard to solution accuracy and iterative number. The local minimizers obtained are shown in Table 1.

Table 1 Results for Example 5.1 with initial point (3, -3)

k	Our algorithm		Algorithm in [19]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	(2.8276,-3.0899)	22.2231	(6.5818e-09,-0.5946)	3.3300
2	(6.7354, 2.3563)	3.7415	(5.7313,-1.8658)	2.1279
3	(1.8974,-0.3005)	3.9293e-15	(1.4513,-4.4244e-08)	0.2264
4			(5.5244,-0.1037)	3.3221e-02
5			(1.0000,-3.0748e-05)	1.3495e-08

Example 5.2. Six-hump back camel function with six local minimizers

$$\begin{aligned}&\min f(x) = 4x_1^2 - 2.1x_1^4 + \tfrac{1}{3}x_1^6 - x_1x_2 - 4x_2^2 + 4x_2^4\\&\text{s.t.} - 3 \le x_1 \le 3, -3 \le x_2 \le 3.\end{aligned}$$

This problem has two global minimizers x^* = (-0.0898, -0.7127) and x^* = (0.0898, 0.7127) with $f(x^*)$ =-1.0316.

Our algorithm successfully finds the global minimizer x^* = (-0.0898, -0.7127) through two iterations, which is better than the results in the paper [19] with regard to iterative number. The local minimizers obtained are shown in Table 2.

Table 2 Results for Example 5.2 with initial point $(3, -3)$

k	Our algorithm		Algorithm in [19]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	(1.6071,-0.5687)	2.1043	(1.6071,-0.5687)	2.1043
2	(-0.0898,-0.7127)	-1.0316	(1.7036,0.7961)	-0.2155
3			(0.0898,0.7127)	-1.0316

Example 5.3. Treccani function with two local minimizers

$$\min f(x) = x_1^4 + 4x_1^3 + 4x_1^2 + x_2^2$$
$$\text{s.t.} -3 \leq x_1 \leq 3, -3 \leq x_2 \leq 3.$$

This problem has two global minimizers $x^* = (0,0)$ and x^* = (-2,0) with $f(x^*) = 0$.

Our algorithm successfully finds an approximate global minimizer x^* = (-2, -7.5024e-09) with (x^*) = 5.6286e-17 through two iterations, which is better than the results in the paper [19] with regard to solution accuracy and iterative number. The local minimizers obtained are shown in Table 3.

Table 3 Results for Example 5.3 with initial point $(2, 2)$

k	Our algorithm		Algorithm in [19]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	(-3.8942e-08,2.4265e-08)	6.6548e-15	(-7.4087e-08,2.3668e-08)	2.2516e-14
2	(-2, -7.5024e-09)	5.6286e-17		

Example 5.4. Three-hump back camel function with three local minimizers

$$\min f(x) = 2x_1^2 - 1.05x_1^4 + \tfrac{1}{6}x_1^6 - x_1x_2 + x_2^2$$
$$\text{s.t.} -3 \leq x_1 \leq 3, -3 \leq x_2 \leq 3.$$

This problem with three local minimizers has the global minimizer $x^* = (0,0)$ and $f(x^*) = 0$.

Our algorithm successfully finds an approximate global minimizer x^* = (4.3837e-09,-6.2778e-09) with $f(x^*)$ = 1.0536e-16 through two iterations. Although the solution accuracy is weaker than the algorithm in the paper [19], it still maintains an efficiency advantage. The local minimizers obtained are shown in Table 4.

Table 4 Results for Example 5.4 with initial point $(1.5, 1.5)$

k	Our algorithm		Algorithm in [19]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	(-1.7476, -0.8738)	0.2986	(-1.7475, -0.8738)	0.2986
2	(4.3837e-09,-6.2778e-09)	1.0536e-16	(6.5624e-08,1.638e-08)	7.8061e-15

Example 5.5. Shubert function with 760 local minimizers

$$\begin{aligned} &\min f(x) = \{\sum_{i=1}^{5} i\cos[(i+1)x_1] + i\} \cdot \{\sum_{i=1}^{5} i\cos[(i+1)x_2] + i\} \\ &\text{s.t.} -10 \leq x_1 \leq 10, -10 \leq x_2 \leq 10. \end{aligned}$$

This problem has 18 global minimizers with $f(x^*) = 0$.

Our algorithm successfully finds the global minimizer x^* = (-1.4251, -0.8003) with $f(x^*)$ = -186.7309 through three iterations, which is better than the results in the paper [19] with regard to iterative number. The local minimizers obtained are shown in Table 5.

Table 5 Results for Example 5.5 with initial point $(1, 1)$

k	Our algorithm		Algorithm in [19]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	(3.0000, 3.0000)	0.0509	(2.2992, 1.8057)	-7.9834
2	(2.7859, -3.0000)	-9.5371	(-2.5109, -3.0000)	-12.9624
3	(-1.4251, -0.8003)	-186.7309	(-1.4251,-3.0000)	-46.5027
4			(-1.4251, -0.8003)	-186.7309

Example 5.6. n-dimensional Sine-square II function with 10^n local minimizers

$$\begin{aligned} &\min f(x) = \tfrac{\pi}{n}\{10sin^2\pi x_1 + \sum_{i=1}^{n-1} [(x_i-1)^2(1+10\sin^2\pi x_{i+1})] + (x_n-1)^2\} \\ &\text{s.t.} -10 \leq x_i \leq 10, i = 1, 2, \ldots, n. \end{aligned}$$

This problem has the global minimizer $x^* = (1, \ldots, 1)$ with $f(x^*) = 0$.

Example 5.7. n-dimensional Ackley function with multiple local minimizers

$$\begin{aligned} &\min f(x) = -20epx(-0.2\sqrt{\tfrac{1}{n}\sum_{i=1}^{n} x_i^2}) - \exp(\tfrac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)) + 20 + e \\ &\text{s.t.} -32.768 \leq x_i \leq 32.768, i = 1, 2, \ldots, n. \end{aligned}$$

This problem has the global minimizer $x^* = (0, \ldots, 0)$ with $f(x^*) = 0$.

Example 5.8. n-dimensional Rastrigin function with multiple local minimizers

$$\begin{aligned} &\min f(x) = 10n + \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i)) \\ &\text{s.t.} -5.12 \leq x_i \leq 5.12, i = 1, 2, \ldots, n. \end{aligned}$$

This problem has the global minimizer $x^* = (0, \ldots, 0)$ with $f(x^*) = 0$.

Examples 5.6-5.8 are solved for $n = 10, 30, 50$. The calculation results are shown in Table 6.

Since the local minimizers of the filled function in this paper are those of the objective function, the new filled function method can avoid solving the objective function again, which is the significant difference from the filled function algorithm in the paper [19], and also is the main contribution of this paper. Hence, the two filled function algorithms are further compared in terms of the function evaluation times about the objective function and the filled function, the running time as well as the total local minimizers of the filled function and the objective function obtained when the algorithm terminates. The comparative results are displayed in Table 6.

The experimental results listed in Tables 1-6 demonstrate that the constructed filled function algorithm in this paper can effectively address the two-dimensional Examples 5.1-5.5, as well as the filled function proposed in the paper [19]. For the multi-dimensional Examples 5.6-5.8, our algorithm can find the optimal or approximate optimal solution, while the algorithm in the paper [19] plunges into the local solution. Furthermore, when the dimensionality of the problems increases, our algorithm shows better ability to jump out of local minimizers than the algorithm in the paper [19]. Overall, our algorithm is significantly superior to the optimization ability of the filled function in the paper [19].

Table 6 Comparison results of Examples 5.1-5.8

$No.$	n	Our algorithm					Algorithm in [19]				
		c_k	c_f	c_P	$f(x_k^*)$	$t(s)$	c_k	c_f	c_P	$f(x_k^*)$	$t(s)$
5.1	2	3	55	1876	$4.0045e$-16	1.0865	9	237	3125	$2.1282e$-09	3.7202
5.2	2	2	32	1002	-1.0316	1.5611	5	106	1563	-1.0316	5.6460
5.3	2	2	24	2358	$7.7902e$-17	0.7542	1	32	3200	$3.5996e$-15	2.6705
5.4	2	2	33	772	$3.1097e$-16	1.3363	3	87	1674	$1.0130e$-13	1.7751
5.5	2	3	46	2186	-186.7309	0.6860	7	206	3125	-186.7309	4.6182
5.6	10	2	509	2554	$4.4940e$-15	5.5445	7	800	5625	$1.7128e$-13	15.0631
	30	3	1903	4431	$2.3824e$-15	20.9858	9	3845	9445	$1.8859e$-11	52.5521
	50	6	5390	14852	$2.2082e$-13	50.2716	9	8158	29656	$7.9324e$-02	160.1634
5.7	10	2	850	2540	$6.4049e$-11	1.9219	3	481	2604	2.0133	3.7657
	30	2	903	4431	$1.2454e$-10	7.7215	9	1641	5935	3.9346	9.2858
	50	2	894	12078	$9.9605e$-11	11.0283	11	1994	47914	10.8371	15.9792
5.8	10	3	707	2253	0	2.9465	15	2572	7612	0	5.4788
	30	4	1807	5640	0	15.9778	13	7083	16825	19.8991	20.0371
	50	7	4738	16668	0	27.3822	13	6762	56630	16.9142	50.1178

6 Conclusions

A new filled function without parameters and ill-conditioned term is constructed for unconstrained global optimization problems in this paper. Theoretical analyses show that the filled function conquers some disadvantages of existing filled functions. In the region where the feasible points are better than the current local minimizer, the filled function has the same local minimizers as the objective function, and its function value is smaller than those in the region where the feasible points is worse than the current local minimizer. Then a new filled function algorithm is given, which simplifies the iterative process of traditional filled function algorithm and is compared with existing ones in finding global minimizer of some global optimization problems. The numerical results reveal the superiority of the presented filled function for solving unconstrained global optimization problems.

References

[1] M.S. Daoud, M. Shehab, H.M. Al-Mimi, L. Abualigah, R.A. Zitar and M.K.Y. Shambour, Gradient-based optimizer (GBO): A review, theory, variants, and applications, *Archives of Computational Methods in Engineering* 30 (2023) 2431–2449.

[2] F. Dong, J. Zhao, J. Zhao, J. Song, J. Chen and Z. Zheng, Robust optimization of PMLSM based on a new filled function algorithm with a sigma level stability convergence criterion, *IEEE Transactions on Industrial Informatics* 17 (2021) 4743–4754.

[3] Y. Gao, Y. Yang and M. You, A new filled function method for global optimization, *Applied Mathematics and Computation* 268 (2015) 685–695.

[4] R.P. Ge, A filled function method for finding a global minimizer of a function of several variables, *Mathematical Programming* 46 (1990) 191–204.

[5] R.P. Ge and Y.F. Qin, A class of filled functions for finding global minimizers of a function of several variables, *Journal of Optimization Theory and Applications* 54 (1987) 241–252.

[6] Y.Y. Huang, C.H. Liu and Y.L. Shang, Inexact operator splitting method for monotone inclusion problems, *Journal of the Operations Research Society of China* 9 (2021) 273–306.

[7] W. Jiang, D. Wu, W. Dong, J. Ding, Z. Ye, P. Zeng and Y. Gao, Design and validation of a nonparasitic 2R1T parallel hand-held prostate biopsy robot with remote center of motion, *Journal of Mechanisms and Robotics* 16 (2023): 051009.

[8] H.W. Jiao and Y.L. Shang, Two-level linear relaxation method for generalized linear fractional programming, *Journal of the Operations Research Society of China* 11 (2023) 569–594.

[9] H. Lin, Y. Gao, X. Wang and S. Su, A filled function which has the same local minimizer of the objective function, *Optimization Letters* 13 (2019) 761–776.

[10] H. Lin, Y. Wang, Y. Gao and X. Wang, A filled function method for global optimization with inequality constraints, *Computational and Applied Mathematics* 37 (2018) 1524–1536.

[11] A.F. Ling, C.X. Xu and F.M. Xu, A discrete filled function algorithm for approximate global solutions of max-cut problems, *Journal of Computational and Applied Mathematics* 220 (2008) 643–660.

[12] C.H. Liu, Y. L. Shang and P. Han, A new infeasible-interior-point algorithm for linear programming over symmetric cones, *Acta Mathematicae Applicatae Sinica, English Series* 33 (2017) 771–788.

[13] X. Liu, Finding global minima with a computable filled function, *Journal of Global optimization* 19 (2001) 151–161.

[14] H.C. Lu, H.Y. Tseng and S.W. Lin, Double-track particle swarm optimizer for nonlinear constrained optimization problems, *Information Sciences* 622 (2023) 587–628.

[15] S. Ma and Y. Gao, Parameter-free filled function method for nonlinear integer program, *Journal of Industrial and Management Optimization* 19 (2023) 6835–6855.

[16] S. Ma, Y. Gao, B. Zhang and W. Zuo, A new nonparametric filled function nethod for integer programming problems with constraints, *Mathematics* 10 (2022): 734.

[17] R. Pandiya, W. Widodo and I. Endrayanto, Non parameter-filled function for global optimization, *Applied Mathematics and Computation* 391 (2021): 125642.

[18] C.J. Price, B.L. Robertson and M. Reale, A hybrid Hooke and Jeevesdirect method for non-smooth optimization, *Advanced Modeling and Optimization* 11 (2009) 43–61.

[19] D. Qu, J Li, Y. Shang, D. Wu and Z. Fang, A novel convergent filled function algorithm for multi-dimensional global optimization, *Optimization* 72 (2023) 2547–2560.

[20] Y. L. Shang and L. S. Zhang, A filled function method for finding a global minimizer on global integer optimization, *Journal of Computational and Applied Mathematics* 181 (2005) 200–210.

[21] Y.L. Shang, D.G. Pu and A.P. Jiang, Finding global minimizer with one-parameter filled function on unconstrained global optimization, *Applied Mathematics and Computation* 191 (2007) 176–182.

[22] Y.L. Shang, Z.F. Jin and D.G Pu, A new filter QP-free method for the nonlinear inequality constrained optimization problem, *Journal of Inequalities and Applications* 2018 (2018): 278.

[23] G.L. Sun, Y.L. Shang and R.X. Zhang, An efficient and robust improved whale optimization algorithm for large scale global optimization problems, *Electronics* 11 (2022): 1475.

[24] G.L. Sun, Y.L. Shang, K.H. Yuan and H.M. Gao, An improved whale optimization algorithm based on nonlinear parameters and feedback mechanism, *International Journal of Computational Intelligence Systems* 15 (2022): 38.

[25] Y.J. Wang and J.S. Zhang, A new constructing auxiliary function method for global optimization, *Mathematical and Computer Modelling* 47 (2008) 1396–1410.

[26] Z.Y. Wang and Y.J. Yang, Novel global optimization algorithm with a space-filling curve and integral function, *Journal of the Operations Research Society of China* 9 (2021) 619-640.

[27] X. Wu, Y. Wang and N. Fan, A new filled function method based on adaptive search direction and valley widening for global optimization, *Applied Intelligence* 51 (2021) 6234–6254.

[28] Q. Yan, W. Chen and X. Yang, A novel one-parameter filled function method with an application to pathological analysis, *Optimization Letters* 18 (2024) 803–824.

[29] Y. Yang and F. Bai, An integral function and vector sequence method for unconstrained global optimization, *Journal of Global Optimization* 50 (2011) 293–311.

[30] Y. Yang and Y. Shang, A new filled function method for unconstrained global optimization, *Applied Mathematics and Computation* 173 (2006) 501–512.

[31] Y. Zhang, Y. Xu and L. Zhang, A filled function method applied to nonsmooth constrained global optimization, *Journal of Computational and Applied Mathematics* 232 (2009) 415–426.

[32] W. Zhu, A class of filled functions for box constrained continuous global optimization, *Applied Mathematics and Computation* 169 (2005) 129–145.

[33] W. Zhu and G. Lin, A dynamic convexized method for nonconvex mixed integer nonlinear programming, *Computers & Operations Research* 38 (2011) 1792–1804.

Manuscript received 28 May 2024
revised 21 July 2024
accepted for publication 13 August 2024

Guanglei Sun
School of Mathematics and Statistics
Henan University of Science and Technology
Luoyang 471023, China
E-mail address: guangleisun03@163.com

Youlin Shang
School of Mathematics and Statistics
Henan University of Science and Technology
Luoyang 471023, China
E-mail address: mathshang@sina.com

Xiaoqiang Wang
School of Mechatronics Engineering
Henan University of Science and Technology
Luoyang 471023, China
E-mail address: wang_xq2002@163.com

Roxin Zhang
Department of Mathematics and Computer Science
Northern Michigan University
Marquette, MI 49855, USA
E-mail address: rzhang@nmu.edu