



H_∞ FILTER ANALYSIS AND DESIGN FOR DELAY SYSTEMS WITH POLYTOPIC-TYPE UNCERTAINTIES*

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This paper is dedicated to Professor Kok Lay Teo' 70th birthday.

Abstract: This paper studies the H_∞ filter design problem for delay systems with polytopic-type uncertainties. A novel Lyapunov-Krasovkii-based H_∞ filter design technique is developed to ensure that both the filter error dynamic system is asymptotically stable and the given H_∞ performance is satisfied. In addition, based on these H_∞ performance analysis results, an efficient filter is designed by using sufficient conditions expressed in form of linear matrix inequalities (LMI), which can be efficiently solved by convex optimization techniques.

Key words: *polytopic-type, uncertain time-varying delay systems, H_∞ filter design.*

Mathematics Subject Classification: *93B52, 93B36.*

1 Introduction

Over the past few decades, uncertain systems have attracted considerable attention and among them time-varying delay systems with polytopic-type uncertainties have gradually become one of the hottest research topics. For example, in [1], the stability problem of time-varying delay systems with polytopic-type uncertainties is investigated using the LMI method. In [2], delay-dependent robust stability of uncertain delay systems is discussed, and moreover, the H_∞ filter design method for discrete-time delay systems is established to deal with polytopic-type uncertainties in [3, 4]. In order to enhance robustness of the filters, a robust H_∞ filter is designed for stochastic uncertain systems in [5], and delay-dependent robust H_∞ filter design is discussed in [6] for uncertain linear systems with time-varying delays. For more results on delay-related design and control, see [7–9] and the references therein.

Inspired by these previous results, this paper will discuss the H_∞ filter design problem for a time-varying delay system with polytopic-type uncertainties. Firstly, by establishing a new Lyapunov function, sufficient conditions for the existence of H_∞ filters are developed. These conditions are obtained by estimating the upper limit value during the Lyapunov function

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derivative's calculation process. Then, the H_∞ filter of this system can be obtained by solving a series of LMIs.

In this paper, the notation is quite standard. The superscripts “-1” and “T” stand for the inverse and transpose of the matrix respectively; R^n is defined as the n -dimensional Euclidean space; $R^{n \times m}$ is the set of all $n \times m$ matrices; $P > 0$ indicates that P is positive definite; I is an identity matrix with appropriate dimension; $\text{diag}\{\dots\}$ is defined as the block diagonal matrix; and “*” is used to indicate the symmetric item of a symmetric matrix.

2 Problem Formulation

Consider the following time-varying delay system with polytopic-type uncertainties,

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d(t)) + Bw(t), \\ y(t) = C_y x(t) + D_y w(t), \\ z(t) = C_z x(t) + D_z w(t), \\ x(t) = \varsigma(t), t \in [-\tau, 0], \end{cases} \quad (2.1)$$

where $x(t) \in R^n$ is the state variable; $y(t) \in R^m$ is the measurable output variable; $w(t) \in R^q$ is the output disturbance satisfying $w(k) \in L_2[0, +\infty)$; and $z(t) \in R^p$ is the output signal to be estimated. $A, A_d, C_y, D_y, C_z, D_z$ are with appropriate dimensions, and $d(t)$ is the time-varying delay satisfying the following conditions,

$$0 \leq d(t) \leq \tau \quad (2.2)$$

and

$$\dot{d}(t) \leq \mu \leq 1. \quad (2.3)$$

Define system matrices $A, A_d, C_y, D_y, C_z, D_z$ that belongs to the following polytopic-type uncertain region,

$$\chi = [A, A_d, C_y, D_y, C_z, D_z] \in \Xi, \quad (2.4)$$

where Ξ is the real convex polytopic domain

$$\Xi := \left[\chi(\lambda) = \sum_{i=1}^q \lambda_i \chi_i; \sum_{i=1}^q \lambda_i = 1, \lambda_i \geq 0 \right]. \quad (2.5)$$

The q vertices of the polytopic can be described as:

$$\chi_i = [A_i, A_d^{(i)}, C_y^{(i)}, D_y^{(i)}, C_z^{(i)}, D_z^{(i)}].$$

Consider the following filter system of system (2.1),

$$\begin{cases} \dot{\hat{x}}(t) = A_f \hat{x}(t) + B_f y(t), \\ \hat{z}(t) = C_f \hat{x}(t) + D_f y(t), \end{cases} \quad (2.6)$$

where A_f, B_f, C_f, D_f are given filter matrices with appropriate dimension. Our objective is to design an asymptotically stable linear filter system (2.6) for system (2.1).

Define $\eta = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}$, and let $e(t) = z(t) - \hat{z}(t)$. Then, we can obtain the following filter dynamic error system for system (2.1):

$$\begin{cases} \dot{\eta}(t) = A_\varepsilon \eta(t) + A_{d\varepsilon} \eta(t - d(t)) + B_\varepsilon w(t), \\ e(t) = C_\varepsilon \eta(t) + D_\varepsilon w(t), \\ \eta(t) = [\zeta^T(t) \ 0]^T, t \in [-\tau, 0]. \end{cases} \tag{2.7}$$

Here,

$$A_\varepsilon = \begin{bmatrix} A & 0 \\ B_f C_y & A_f \end{bmatrix} \quad A_{d\varepsilon} = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix} \quad B_\varepsilon = \begin{bmatrix} B \\ B_f D_y \end{bmatrix} \quad C_\varepsilon = \begin{bmatrix} C_z - D_f C_y & -C_f \end{bmatrix} \\ D_\varepsilon = D_z - D_f D_y$$

For the given $\tau, \mu, \gamma > 0$, if we can find a full-order linear time-invariant filter for system (2.1), then for any delay that satisfies (2.2) and (2.3), it will hold that (i) the filter error system (2.7) is asymptotically stable; and (ii) under the initial conditions, for all non-zero $w(t) \in L_2[0, +\infty]$ and $\lambda > 0$, the filter error system (2.7) has the H_∞ properties:

$$\| e \|_2 \leq \gamma \| \Xi \|_2 .$$

3 H_∞ Performance Analysis

The following theorem gives a sufficient condition for the existence of the filter system (2.6).

Theorem 3.1. Let $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$, $Q_1 > 0$, $Q_2 > 0$, $Z > 0$, $R > 0$, $X = [X_{ij}]_{5 \times 5} \geq 0$,

$$M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{bmatrix}, \quad H = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix}, \quad Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix} . \quad \text{For the given } \tau > 0, \mu < 1 \text{ and } \gamma > 0,$$

the augmented system (2.7) is asymptotically stable while the time delay satisfies conditions (2.2) and (2.3), and it also satisfies the H_∞ performance: $\| e \|_2 \leq \gamma \| \Xi \|_2$ if the following LMIs are valid,

$$\phi^{(i)} = \begin{bmatrix} \Xi^{(i)} + \tau X & \Gamma_{(i)}^T & \tau \bar{A}_{(i)}^T Z & Y^T R^{-1} & \bar{A}_{(i)}^T R \\ * & -I & 0 & 0 & 0 \\ * & * & -\tau Z & 0 & 0 \\ * & * & * & -R^{-1} & 0 \\ * & * & * & * & -R \end{bmatrix} < 0, \quad \forall i = 1, 2, \dots, q. \tag{3.1}$$

$$\gamma_1^{(i)} = \begin{bmatrix} X & M + H \\ * & Z \end{bmatrix} \geq 0, \quad \forall i = 1, 2, \dots, q. \tag{3.2}$$

$$\gamma_2^{(i)} = \begin{bmatrix} X & 0 \\ * & 0 \end{bmatrix} \geq 0, \quad \forall i = 1, 2, \dots, q. \tag{3.3}$$

where $\Xi^{(i)} = \begin{bmatrix} \Xi_{jk}^{(i)} \end{bmatrix}_{5 \times 5}$, $\bar{A}^{(i)} = \begin{bmatrix} A^{(i)} & 0 & A_d^{(i)} & 0 & B^{(i)} \end{bmatrix}$, $\Gamma^{(i)} = \begin{bmatrix} C_z^{(i)} - D_f C_y^{(i)} & -C_f & 0 & 0 & D_z^{(i)} - D_f D_y^{(i)} \end{bmatrix}$, $\Xi_{11}^{(i)} = A_{(i)}^T P_1 + P_1 A^{(i)} + C_y^{(i)T} B_f^T P_2^T + P_2 B_f C_y^{(i)} + Q_1 + Q_2 + M_1 + M_1^T + Y_1 A^{(i)} + A_{(i)}^T Y_1^T + H_1^T + H_1$, $\Xi_{12}^{(i)} = A_{(i)}^T P_2 + C_y^{(i)T} B_f^T P_3 +$

$$\begin{aligned}
P_2 A_f + M_2^T + A_{(i)}^T Y_2^T + H_2^T, \Xi_{13}^{(i)} &= P_1 A_d^{(i)} - M_1 + M_3^T + H_3^T + A_{(i)}^T Y_3^T + Y_1 A_d^{(i)}, \Xi_{14}^{(i)} = M_4^T - \\
H_1 + H_4^T + A_{(i)}^T Y_4^T, \Xi_{15}^{(i)} &= P_1 B^{(i)} + P_2 B_f D_y^{(i)} + M_5^T + H_5^T + A_{(i)}^T Y_5^T + Y_1 B, \Xi_{22}^{(i)} = A_f^T P_3 + P_3 A_f, \\
\Xi_{23}^{(i)} &= -M_2 + Y_2 A_d^{(i)}, \Xi_{24}^{(i)} = -H_2, \Xi_{25}^{(i)} = P_2 B^{(i)} + P_3 B_f D_y^{(i)} + Y_2 B^{(i)}, \Xi_{33}^{(i)} = -(1 - \mu) Q_1 - \\
M_3^T - M_3 + A_d^{(i)} Y_3^T + Y_3 A_d^{(i)}, \Xi_{34}^{(i)} &= -M_4^T - H_3 + A_d^{(i)T} Y_4^T, \Xi_{35}^{(i)} = -M_5^T + Y_3 B^{(i)} + A_d^{(i)T} H_5^T, \\
\Xi_{44}^{(i)} &= -Q_2 + H_4^T - H_4, \Xi_{45}^{(i)} = -H_5^T + Y_4 B^{(i)}, \Xi_{55}^{(i)} = -\gamma^2 I + Y_5 B^{(i)} + B_{(i)}^T Y_5^T.
\end{aligned}$$

Proof. We choose the following Lyapunov function candidate,

$$\begin{aligned}
V(x_t) &:= \eta^T(t) P \eta(t) + \int_{t-d(t)}^t x^T(s) Q_1 x(s) ds + \int_{t-\tau}^t x^T(s) Q_2 x(s) ds \\
&\quad + \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s) Z \dot{x}(s) ds d\theta.
\end{aligned} \tag{3.4}$$

By taking the derivative of (3.4) along the system (2.7), we can obtain

$$\begin{aligned}
\dot{V}(x_t) &:= 2\eta^T(t) P \dot{\eta}(t) + x^T(t) (Q_1 + Q_2) x(t) - (1 - \dot{d}(t)) x^T(t - d(t)) Q_1 x(t - d(t)) \\
&\quad - x^T(t - \tau) Q_2 x(t - \tau) + \tau \dot{x}^T(t) Z \dot{x}(t) - \int_{t-\tau}^t \dot{x}^T(s) Z \dot{x}(s) ds \\
&\leq 2\eta^T(t) P \dot{\eta}(t) + x^T(t) (Q_1 + Q_2) x(t) - (1 - \mu) x^T(t - d(t)) Q_1 x(t - d(t)) \\
&\quad - x^T(t - \tau) Q_2 x(t - \tau) + \tau \dot{x}^T(t) Z \dot{x}(t) - \int_{t-\tau}^t \dot{x}^T(s) Z \dot{x}(s) ds
\end{aligned} \tag{3.5}$$

By applying the Leibniz-Newton theory, we get

$$2\xi^T(t) M[x(t) - x(t - d(t)) - \int_{t-d(t)}^t \dot{x}(s) ds] = 0 \tag{3.6}$$

$$2\xi^T(t) H[x(t) - x(t - \tau) - \int_{t-\tau}^t \dot{x}(s) ds] = 0 \tag{3.7}$$

$$\tau \xi^T(t) X \xi(t) - \int_{t-d(t)}^t \xi^T(t) X \xi(t) ds - \int_{t-\tau}^{t-d(t)} \xi^T(t) X \xi(t) ds = 0 \tag{3.8}$$

and

$$2\xi^T(t) Y [Ax(t) + A_d x(t - d(t)) + Bw(t) - \dot{x}(t)] = 0 \tag{3.9}$$

Then, the following inequality will be held,

$$-2\xi^T(t) Y \dot{x}(t) \leq \xi^T(t) Y R^{-1} Y^T \xi(t) + \xi^T(t) \bar{A}^T R \bar{A} \xi(t), \tag{3.10}$$

where $\xi^T(t) = [x^T(t) \quad \hat{x}^T(t) \quad x^T(t - d(t)) \quad x^T(t - \tau) \quad w^T(t)]$. After adding the left side of (3.6) to (3.9) and applying (3.10), we obtain

$$\begin{aligned}
\dot{V}(x_t) - \gamma^2 w^T(t) w(t) + e^T(t) e(t) &\leq \xi^T(t) [\Xi + \tau X + \Gamma^T \Gamma + \tau \bar{A}^T Z \bar{A} + Y^T R^{-1} Y + \bar{A}^T R \bar{A}] \xi(t) \\
&\quad - \int_{t-\tau}^t \xi^T(t, s) \gamma_1 \xi(t, s) ds - \int_{t-\tau}^{t-d(t)} \xi^T(t, s) \gamma_2 \xi(t, s) ds,
\end{aligned}$$

where $\bar{A} = [A \ 0 \ A_d \ 0 \ B]$, $\Gamma = [C_z - D_f C_y \quad -C_f \ 0 \ 0 \ D_z - D_f D_y]$, $\xi^T(t, s) = [\xi^T(t) \quad \dot{x}(s)]$. According to Schur complement lemma, (3.1) shows that across the entire

uncertain region Ξ , there is $\Xi + \tau X + \Gamma^T \Gamma + \tau \bar{A}^T Z \bar{A} + Y^T R^{-1} Y + \bar{A}^T R \bar{A} < 0$. Based on (3.1) to (3.3), we can ensure that $\dot{V}(x_t) - \gamma^2 w^T(t)w(t) + Ze^T(t)Ze(t) < 0$ is valid across the entire uncertain region Ξ . It also indicates that the augmented system (2.7) is asymptotically stable when $w(t) = 0$.

Under the zero initial condition $V(x_t) |_{t=0} = 0$, we obtain

$$\int_0^\infty [e^T(t)e(t) - \gamma^2 w^T(t)w(t)] dt \leq V(x_t) |_{t=0} - V(x_t) |_{t \rightarrow \infty} < 0.$$

Therefore, $\|e\|_2 \leq \gamma \|w\|_2$ holds. The proof is complete. □

Note: The Lyapunov function candidate in Theorem 1 is constructed based on two different Lyapunov functions in [1] and [6]. Inequality (3.10) is obtained using the Leibniz-Newton principle and [2].

4 Filter Design

In this section, the method to obtain the filter parameter variables $\{A_f, B_f, C_f, D_f\}$ will be developed based on Theorem 1. By solving a series of linear matrix inequalities, these parameter variables $\{A_f, B_f, C_f, D_f\}$ can be obtained. Then, we can establish the following theorem.

Theorem 4.1. *For the given $\tau > 0, \gamma > 0$ and $\mu > 0$, the H_∞ filter model (2.5) for system (2.1) is valid, if there are matrices $P_1 > 0, T > 0, Q_1 > 0, Z > 0, R > 0, N_j > 0, j = 1, 2, 3, 4$;*

$$\hat{X} = [\hat{X}_{ij}]_{5 \times 5} \geq 0, \text{ and } M = \begin{bmatrix} M_1 \\ \hat{M}_2 \\ M_3 \\ M_4 \\ M_5 \end{bmatrix}, H = \begin{bmatrix} H_1 \\ \hat{H}_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix}, Y = \begin{bmatrix} Y_1 \\ \hat{Y}_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix},$$

and the appropriate-dimensional matrix enables

$$\begin{bmatrix} P_1 - T & 0 \\ 0 & T \end{bmatrix} > 0 \tag{4.1}$$

$$\Theta^{(i)} = \begin{bmatrix} \Xi^{(i)} + \tau X & \Gamma^{(i)T} & \tau \bar{A}^{(i)T} Z & H^T R^{-1} & \bar{A}^{(i)T} R \\ * & -I & 0 & 0 & 0 \\ * & * & -\tau Z & 0 & 0 \\ * & * & * & -R^{-1} & 0 \\ * & * & * & * & -R \end{bmatrix} < 0, \quad \forall i = 1, 2, \dots, q \tag{4.2}$$

$$\Pi_1^{(i)} = \begin{bmatrix} \hat{X} & M + H \\ * & Z \end{bmatrix} \geq 0, \quad \forall i = 1, 2, \dots, q \tag{4.3}$$

$$\Pi_2^{(i)} = \begin{bmatrix} \hat{X} & Y \\ * & 0 \end{bmatrix} \geq 0, \quad \forall i = 1, 2, \dots, q \tag{4.4}$$

where $\Xi^{(i)}, \bar{A}^{(i)}, \Gamma^{(i)}$ are the same with those in Theorem 1.

Proof. We will follow a similar proof procedure of Theorem 3 in [6] and it will be omitted here. The proof is complete. □

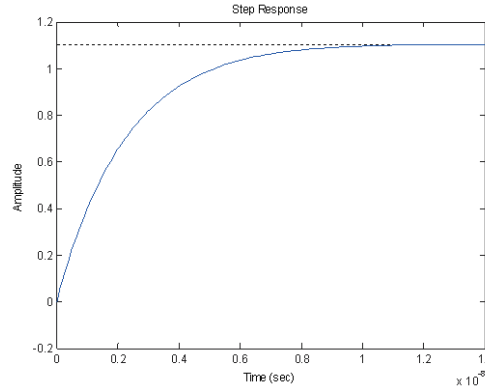


Figure 1: Step response curve of the filtering system when $\mu = 0.5, \gamma = 0.6123, \tau = 1$.

5 Illustrative Examples

This section provides two examples to demonstrate the effectiveness of the criteria presented in this paper.

Example 1. Consider the following system with polytopic-type uncertainties

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 2 \\ -1 & -3 \end{bmatrix} x(t) + \begin{bmatrix} -0.1 & 0 \\ 0.2 & -0.1 \end{bmatrix} x(t - d(t)) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + w(t) \\ z(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t) \end{cases} \quad (5.1)$$

When $\mu = 0.5, \gamma = 0.6123, \tau = 1$, by solving LMIs (4.2)-(4.4), we can get

$$A_f = (1.0e + 008) * \begin{bmatrix} -2.4066 & -0.1800 \\ -0.18001 & -4.2068 \end{bmatrix}, \quad B_f = (1.0e + 008) * \begin{bmatrix} -0.6874 \\ -2.0607 \end{bmatrix}, \\ C_f = \begin{bmatrix} 0.3242 & -2.4717 \end{bmatrix}, \quad D_j = -2.7072e - 005.$$

It can be clearly seen in Figure 1 that when $\mu = 0.5, \gamma = 0.6123, \tau = 1$ the H_∞ filtering system is asymptotically stable. In addition, when $\mu = 0.9, \gamma = 1, \tau = 0.6872$, by solving LMIs (4.2)-(4.4), we have

$$A_f = (1.0e + 008) * \begin{bmatrix} -1.6645 & -0.1706 \\ -0.1706 & -3.1019 \end{bmatrix}, \quad B_f = (1.0e + 008) * \begin{bmatrix} -0.4458 \\ -1.3441 \end{bmatrix}, \\ C_f = \begin{bmatrix} 0.0507 & -2.3419 \end{bmatrix}, \quad D_j = -2.7184e - 004.$$

Clearly, the H_∞ filtering system is asymptotically stable as shown in Figure 2 .

Example 2: Consider the following time-varying delay system

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -0.7 + \rho(t) \end{bmatrix} x(t) + \begin{bmatrix} -1 & -1 + \delta(t) \\ -2 & -1 \end{bmatrix} x(t - d(t)) + \begin{bmatrix} 0.5 \\ 2 \end{bmatrix} w(t) \\ y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) + w(t) \\ z(t) = \begin{bmatrix} 2 & 1 \end{bmatrix} x(t) \end{cases}$$

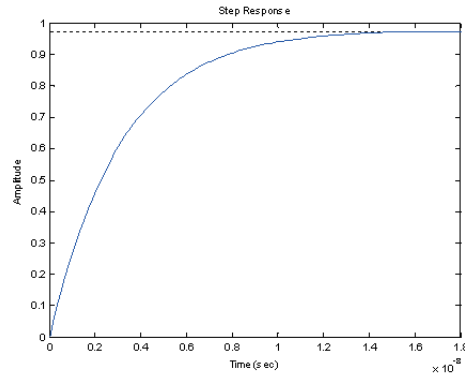


Figure 2: Step response curve of the filtering system when $\mu = 0.9, \gamma = 1, \tau = 0.6872$.

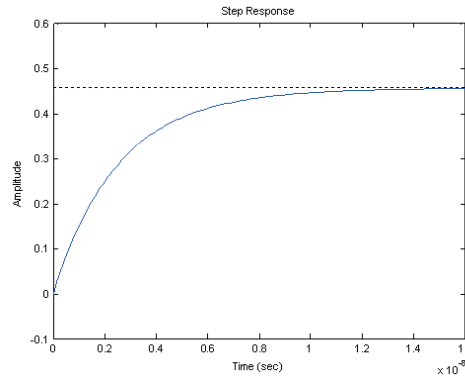


Figure 3: Step response curve of the filtering system when $\mu = 0.4, \gamma = 5, \tau = 0.6393$.

where the uncertain parameters satisfy the conditions $\|\rho(t)\| \leq 0.2$, and $\|\delta(t)\| \leq 0.5$. When $\mu = 0.4, \gamma = 0.5, \tau = 0.6393$, according to LMIs (4.2)-(4.4), we obtain

$$A_f = (1.0e + 007) * \begin{bmatrix} -4.2243 & 0.9668 \\ 0.9668 & -4.8123 \end{bmatrix}, \quad B_f = (1.0e + 007) * \begin{bmatrix} 1.5864 \\ -3.5774 \end{bmatrix},$$

$$C_f = [-1.0073 \quad -0.9616], \quad D_j = -3.0885e - 006.$$

The simulation in Figure 3 shows that the H_∞ filtering system is asymptotically stable.

6 Conclusion

This paper has investigated H_∞ filter design for the time-varying delay system with polytopic-type uncertainties. By applying the Lyapunov-Krasovsii method, the H_∞ filter design expressed by LMIs is provided. The H_∞ filter design for the time-varying delay system with polytopic-type uncertainties will be converted to solving a set of LMIs. Finally, two numerical examples are given to illustrate the usefulness and validity of the results obtained.

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