SHIFT CAPACITY PLANNING FOR NURSING STAFF IN EMERGENCY DEPARTMENT USING MIXED INTEGER PROGRAMMING

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Abstract: Emergency department (ED) management is one of the most challenging fields in health care. In this paper, we present a mathematical optimization model to determine shift scheduling and capacity planning policies for nursing staff in ED. In particular, we develop a mixed integer programming model, where the objective is to minimize the mean deviation of overall workloads over different time intervals of a day. Decision variables for shift capacity planning include the total number of major and minor shifts, shift working hours and nurse capacity for each shift. With ED patient administrative data of a hospital in Singapore, we derive optimal nurse staffing rules under different scenarios of nursing coverage requirements in ED. The obtained results could help plan the shifts of ED nursing staff.

1 Introduction

Emergency department (ED) management is one of the most challenging fields in health care today. ED utilization has been growing rapidly, resulting in overcrowding, consult or admission delays, and increased staff burnout. In the past decade, a number of publications related to ED resource utilization have been published to describe operational strategies for providing quality patient-centered care. Many studies have examined factors associated with ED utilization using regression-based statistical models [24, 29]. Also, a number of studies have investigated patient flows and resource utilization using operations research methods such as queueing theory and discrete event simulation [16, 22].

Emergency nursing takes care of patients of all ages with perceived or actual physical or emotional alterations of health that are undiagnosed or that require further interventions [2]. A report by the World Health Organization [40] states that managing and deploying personnel resources efficiently are key challenges for the healthcare industry in the coming decade due to the increasing demand for care and healthcare providers. In particular, nursing staff is a critical component in heath care services. Studies demonstrate that unattractive schedules, poor practice environments and high workloads are key factors leading to nurse discontentment and a high nursing turnover [10, 39]. On the other hand, proper personnel

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policies or rules were shown to have positive impacts on the nurses' working conditions, which in turn are closely related to the quality of care [39]. These observations motivate hospitals to adopt policies that increasingly accommodate preferences and requests of nursing staff while ensuring suitably qualified staff on duty at the right time [31]. As appropriate staffing and shift scheduling of nursing staff play a role in delivering quality patient care, it is necessary to develop better decision support system for hospital decision makers to manage nursing staff.

Nursing staff management is basically a sequential planning and control process [6, 36], consisting of nurse staffing, shift planning, and allocation. In general, staffing is a strategic and long term manpower planning. Shift scheduling and assignment are to satisfy both the minimum coverage requirements and time-related rules and practices for the nurses and the hospital. Nurse allocation determines the individual nurse's schedule and the timetable for all nurses. In the past decades, a great deal of efforts have been paid to investigate nurse scheduling using various methods from different perspectives. In literature, mathematical optimization approaches, such as linear programming, mixed integer programming, and stochastic programming, had been widely used to study nurse rostering problem [1, 3, 4, 6, 13, 15, 20, 21, 23, 27, 33]. Other methods were proposed for the nurse scheduling problem as well, such as queueing models [12, 16, 37], scheduling models based on hierarchal approach [39]. For a brief overview, the reader is referred to [10].

To deal with nursing staff discontent and frequent turnover, one approach is to introduce adequate shift scheduling policy addressing factors such as nurse working hours, enough breaks and individual preferences [26]. Its importance on patient care was further stressed in [5, 7, 15, 38]. Another approach is to improve the flexibility of nursing staff through cross-utilization [8, 9, 14, 19]. On the other hand, clinical management of patients in emergency department is particularly complex due to differences in patients' urgency, changing conditions of patients and uncertainty of patients' arrivals and diagnoses. Patients may flow into different locations in ED depending on their needs. Consequently, the workload at ED can change dramatically by time of day and day of week. To spread out the workload over the time of a day, some researchers studied the ideal patient to nurse ratio in the general ward setting [17, 28]. However, few studied the case under the ED setting. In particular, there has been an accelerating demand for emergency care targeted to the aging population [30]. It is observed that the ability of staff to cope with patient load not only affects the health outcomes and satisfaction of patients [32], but also affects the morale and wellbeing of the staff [18, 25]. Hence, it is crucial to develop an appropriate nurse staffing model to address main concerns of healthcare providers.

In this paper, our objective is to balance the overall nurse workloads over the time of a day under the current staffing capacity. In particular, we would like to use a mathematical optimization approach to determine optimal staffing rules for shift scheduling and capacity planning at ED. Different from usual treatment, the proposed model also incorporates shift working hours, shift times and total number of shifts as decision variables for planning. For this purpose, an immediate issue is to estimate patient loads (demands) at various areas in ED over different time intervals of a day. In literature, patient arrivals are usually modeled as a Poisson process and patient loads at different areas in ED are estimated using queueing theory with expected service rates (i.e., patient treatment and care) [12]. However, this approach is criticized oftentimes due to the strong arrival assumption and high variability of patient duration in ED by nature. In this study, we use a different manner to assess patient loads by using large scale patient data. Briefly, we study individual patient's movements within various areas in ED starting from triage to leaving ED. For given time of the day, we use a weighted sum of accumulated nursing touch points on average at key areas in ED as

the demand during this time period. We investigate the nurse workload over certain time interval of a day by using the demand and the total number of nursing staff on duty during this period. The proposed model is a deterministic mixed integer programming in which the objective is to minimize the mean deviation of the workloads over different time intervals of a day. The model provides optimal decision rules concerning the total number of shifts to be planned, shift working hours, and the number of nursing staff for each shift. ED managers can use these suggested results in decision making for nurse rostering.

The rest of this paper is organized as follows. The specific problem description on shift capacity planning is stated in Section 2. In Section 3, we formulate the problem as a mixed integer programming model. Section 4 analyzes numerical results of the model under various scenarios with ED patient data. Concluding remarks and future research directions are provided in Section 5.

2 | Problem Description

The hospital in this study is an acute hospital in Singapore of about 1,200 beds. The key functional areas at ED are triage, consult, resuscitation, trolley observation room, fever observation room, and decontamination. In general, when patients arrive at ED, nurses will triage them. Doctors will assess them in consultation rooms. After consultation, patients may proceed to observation rooms, or leave ED. The workload intensity may vary among these areas. For instance, the workload at decontamination area is generally much heavier than at observation room. To evaluate the workload in the entire ED, in this study the demand for nursing staff at different areas is adjusted with the weights, which basically reflect the differences of the workload intensity between the areas. The values of these weights are suggested by emergency department. Our analysis is based on the adjusted demand over various time intervals of a day under consideration.

Nursing staff with different skill sets are deployed at different locations and work in shifts. In this paper, we assume all the nursing staff is fully skilled and time intervals under consideration are referred to half-hourly time periods of a day starting at 00:00. Then, there are total of 48 half-hour intervals of a day in the analysis. Our aim is to balance the overall workload of nursing staff during these time intervals through designing optimal nursing staff deployment policies. In what follows, we first assess the demand and the number of nursing staff on duty at ED over the time intervals.

We say that a patient makes a touch point at some area when the patient arrives at that location. Changes of touch points are captured when staff updates patients' locations in the ED information system. We use the touch points to estimate the demand. Patients could have multiple touch points for a specific location such as consultation areas. In this case, we only use the first touch point to represent the demand at the location. The demand at each time interval is then calculated by the weighted sum of the total touch points of the key locations at ED.

The emergency department of this study operates with three major shifts for nursing staff, i.e., *morning shift*, *afternoon shift* and *night shift*, each with the shift time from 07:00 to 15:30, 13:00 to 21:30, and 21:00 to 07:30, respectively. In addition to the major shifts, there are three minor shifts as well, with the shift time from 09:00 to 17:30, 15:00 to 23:30pm, and 16:00 to 24:00. These three minor shifts operate between the major shifts. In practice, the numbers of nursing staff in minor shifts are relatively small. They are assigned to various locations in ED as extra supporting staff to meet the high patient loads. We take the present complete staff deployment distribution over the 48 half-hour time intervals as the baseline schedule. This together with the estimated demand over time intervals facilitates

the assessment of the ratio of the number of nursing staff to total touch points (or demand) at each half hour, which is referred as a measure of the workload in subsequent analysis.

By analyzing patient administrative data and current nursing staff assignment, it shows that the current workload varies remarkably over the time of a day. In particular, the ratio of nursing staff to touch points is quite high at mid-night or early morning period (03:00 to 06:00) while very low around the noon time (10:00 to 14:30). To deal with the issue, we are interested to develop shift capacity planning model so as to minimize the mean deviation of overall workloads over 48 half-hour time intervals. In particular, we concentrate on the following issues: (i) how many shifts (both major and minor shifts) should we have? (ii) what would be the right shift times? (iii) how many nursing staff should be allocated in each shift? These problems are important for hospital decision makers in capacity planning.

$\boxed{3}$ Shift Capacity Planning Model

In this section, we propose a mathematical optimization model to address the shift capacity planning problem. First, we state some notations used in the model.

Parameters

- C: total capacity of nursing staff

- T_j: j-th time interval of a day, $j = 1, \ldots, J$, where the first time interval starts at 00:00 and the last one ends at 24:00, denoted by an index set $\mathcal{T} := \{T_1, \ldots, T_J\}$

- C_j: minimum number of nursing staff during time interval T_j, $\quad j = 1, \ldots, J$

- d_j: adjusted demand during time interval T_j, $j = 1, \ldots, J$

- $\bar{x}_j$: current number of nursing staff on duty during time interval T_j, $j = 1, \ldots, J$

- ρ: minimum ratio of the number of nursing staff to the demand amongst J time intervals, i.e., $\rho := \min\{\frac{\bar{x}_j}{d_j} : j = 1, \ldots, J\}$

- $\mathcal{K}_1$: index set of major shifts, denoted by $\mathcal{K}_1 := \{\text{morning, afternoon, night}\} = \{M_1, M_2, M_3\}$

- S: maximum number of possible minor shifts of a day, denoted by $\mathcal{K}_2 := \{M_4, \ldots, M_{S+3}\}$

- $\mathcal{K}$: index set of all possible shifts (including both major and minor shifts) of a day, that is, $\mathcal{K} := \mathcal{K}_1 \cup \mathcal{K}_2 = \{M_1, M_2, \ldots, M_{S+3}\}$

- γ: maximum number of minor shifts to be operated in ED

- l_i: lower bound on number of nursing staff in shift $M_i \in \mathcal{K}$, $i = 1, \ldots, S+3$

- u_i: upper bound on number of nursing staff in shift $M_i \in \mathcal{K}$, $i = 1, \ldots, S+3$

- M_0: big positive real number

In practice, nurses' working times may end at some half-hour intervals of a day. Hence, we divide a day into 48 half-hour intervals (i.e., $J = 48$), where $T_1 = [00:00, 00:30], \ldots, T_{48} = [23:30, 24:00]$. Evidently, each shift may cover a number of continual time slots of $\mathcal{T}$. For example, the duration of morning shift (from 07:00 to 15:30) consists of 17 time slots starting from the 15-th time slot and ending by the 31-th slot, namely, $M_1 = \cup_{j=15}^{31} T_j$. At present, the working hours of three major shifts and their shift times are ideal for ED.

Thus, our focus falls on optimal nursing staff capacity in each major shift. On the other hand, nurse staffing in minor shifts, such as shift times and the number of nurses in each shift, is a major concern for hospital managers. We assume that the working hours of all minor shifts are equal to 8.5 hours. Their shift times can start at any starting time of time slots in $\mathcal{T}$. This implies that there is a maximum of 48 possible minor shifts (i.e., $S = 48$) to choose in capacity planning. To streamline the complexity of nursing service process, hospital managers prefer to implement relatively few minor shifts whenever it is possible. To describe this feature, in the model we introduce a parameter γ representing maximum number of minor shifts. Modelers or decision makers can adjust its value in analysis or treat γ as a decision variable. Shift working hours and shift times can be incorporated in the model using a nurse on-duty matrix as discussed later.

The target of the model seeks to reduce the mean deviation of the workloads of the underlying 48 time intervals of a day. Decision variables will include the best combination of minor shifts chosen from 48 minor shift candidates, and the number of nursing staff in each shift. For the former, we consider the optimal number of minor shifts and the respective shift times. To characterize this issue, we denote the set of all feasible minor shifts by $\mathcal{K}_2$, in which each element is defined by a number of time slots T_j and the working hours. For example, M_4 denotes the minor shift starts at 00:00 and ends at 08:30, which can be expressed as $M_4 = \cup_{j=1}^{17} T_j$. In the same way, $M_5 = \cup_{j=2}^{18} T_j$, ..., $M_{48} = T_{48} \cup \{\cup_{j=1}^{16} T_j\}$. Further, we introduce a 0-1 binary variable z_i, $i = 4, \ldots, S + 3$, representing minor shift M_i to be chosen or not, to formulate the requirement on maximum number of minor shifts to be implemented. The decision variables are stated as below.

Decision variables

- x_i: number of nursing staff in shift M_i, $\quad i = 1, \ldots, S + 3$

- y_j: number of nursing staff in time interval T_j, $j = 1, \ldots, J$

- z_i: 0-1 binary variable, $i = 4, \ldots, S + 3$. For each $M_i \in \mathcal{K}_2$,

$$z_i := \begin{cases} 1, & \text{if minor shift } M_i \text{ is selected for capacity planning,} \\ 0, & \text{otherwise.} \end{cases}$$

It is clear that decision variables $x \in \Re^{S+3}$ and $y \in \Re^J$ are interrelated. Note also that we have $J = S = 48$. To address the decisions concerning shift times, we introduce a nursing staff on-duty matrix $A \in \Re^{(S+3) \times J}$, capturing the information whether nurses are on duty or not at each time interval T_j, $j = 1, \ldots, J$, for all possible shifts M_i, $i = 1, \ldots, S + 3$, which might be implemented in emergency department. Specifically, for each $M_i \in \mathcal{K}$ and $T_j \in \mathcal{T}$, each element a_{ij} of A is 0-1 binary number defined as follows.

$$a_{ij} := \begin{cases} 1, & \text{if nursing staff in shift } M_i \text{ is on duty at time interval } T_j, \\ 0, & \text{otherwise.} \end{cases}$$

In general, shift working hours are same for all shifts except for night major shift with a bit longer duration. Let β_1 and β_2 be the respective total shift working hours. We have $\sum_{j=1}^{J} a_{ij} = 2\beta_1$ for each $i \in \{1, \ldots, S + 3, i \neq 3\}$ and $\sum_{j=1}^{J} a_{3j} = 2\beta_2$. In practice, both β_1 and β_2 are of a few feasible choices such as $\beta_1 \in \{8, 8.5, 9\}$ and $\beta_2 \in \{10, 10.5, 11\}$, resulting in 9 different nurse on-duty matrices A altogether. Optimal shift capacity planning rules can be derived by solving proposed optimization model associated with these matrices. Moreover, optimal shift times can be derived according to the solutions of z_i and β_1 or β_2.

Matrix A is thereby an important component in model development which incorporates the information of shift working hours and shift times. In addition, for $j = 1, \ldots, J$, we have

$$y_j = \sum_{i=1}^{S+3} a_{ij} x_i.$$

This linear relationship between x and y can be rewritten in matrix form, i.e., $y = A'x$. For convenience in presentation, we write $z := (z_1, z_2, z_3, z_4, \ldots, z_{S+3})'$ where $z_1 = z_2 = z_3 = 1$. It follows from the notion of 0-1 binary variable that the three major shifts will be chosen in shift capacity planning model by default. Based on the above arguments, we develop a mathematical model for shift capacity planning as follows.

$$\min \quad \frac{1}{J} \sum_{j=1}^{J} \left| \frac{y_j}{d_j} - \frac{1}{J} \sum_{k \in \mathcal{T}} \frac{y_k}{d_k} \right| \tag{3.1}$$

$$\text{s. t.} \quad \mathbf{1}'x \leq C, \tag{3.2}$$

$$l \leq x \leq u, \tag{3.3}$$

$$y = A'x, \tag{3.4}$$

$$y_j \geq \rho d_j, \quad j = 1, \ldots, J, \tag{3.5}$$

$$y_j \geq C_j, \quad j = 1, \ldots, J, \tag{3.6}$$

$$z_1 = z_2 = z_3 = 1, \tag{3.7}$$

$$\sum_{i=4}^{S+3} z_i \leq \gamma, \tag{3.8}$$

$$x_i \leq M_0 z_i, \quad i = 4, \ldots, S + 3, \tag{3.9}$$

$$z_i \leq x_i, \quad i = 4, \ldots, S + 3, \tag{3.10}$$

$$x_i \in Z^+, \ z_i \in \{0, 1\}, \ i = 1, \ldots, S + 3, \ y_j \in Z^+, \ j = 1, \ldots, J. \tag{3.11}$$

Here, $\mathbf{1} \in \Re^{S+3}$ with all elements being ones. Model (3.1) - (3.11) is a nonlinear programming problem with a nonsmooth objective function and integer decision variables subject to a system of linear equalities and inequalities. The objective function (3.1) is the mean deviation of workloads, i.e., mean of the distances of each workload, $\frac{y_j}{d_j}$, $j = 1, \ldots, J$, from their mean, as a measure to smooth the current unbalanced workloads over the time intervals of a day. Constraint (3.2) is the usual capacity budget, that is, the total number of nursing staff assigned to all shifts should not exceed the predetermined capacity. Constraint (3.3) sets lower and upper bounds on the number of nursing staff in each shift. The number of nursing staff being on duty at specific time slot is expressed in (3.4). Constraint (3.5) indicates that the obtained ratios of the workload should be no worse off the current minimum ratio. To meet necessary nursing needs for patient treatment and care, there is a minimum capacity on nursing staff for each time slot of a day. This requirement is stated in (3.6). Constraints (3.8) - (3.10) limit the selection of minor shifts from S possible minor shift candidates based on the idea of big M method in linear programming. The current three major shifts are feasible and commonly adopted in emergency departments. Hence, the total number of major shifts is not considered as a variable in the model. Constraint (3.11) are standard integer requirements on the decision variables.

Note that model (3.1) - (3.11) is very general and flexible. Decision makers can determine optimal manpower planning rules according to their specific requirements and practical considerations through setting different values of the associated parameters in the model.

For example, if managers do not want to operate any minor shifts in ED, they can simply set the value of γ to be zero. The model will force z_i, $i = 4, \ldots, S + 3$, to be zero. On the other hand, if there is no specific requirement on the total number of minor shifts, one may just set γ to be a large number, such as $\gamma = 50$. In this case, the associated model will provide an optimal number of minor shifts for planning. This information could be used as a reference for managers in decision making. In addition, our current study is based on half-hourly analysis. Decision makers or modelers can similarly consider the situation of any arbitrary time interval of interest following the proposed framework.

The nonsmoothness of the objective function makes the model difficult to solve. To deal with this obstacle, we introduce additional variables, w_j, such that $w_j = |\frac{y_j}{d_j} - \frac{1}{J} \sum_{k=1}^{J} \frac{y_k}{d_k}|$, $j = 1, \ldots, J$. Accordingly, we add the following set of inequality constraints to the model.

$$w_j \geq \frac{y_j}{d_j} - \frac{1}{J} \sum_{k=1}^{J} \frac{y_k}{d_k}, \quad w_j \geq \frac{1}{J} \sum_{k=1}^{J} \frac{y_k}{d_k} - \frac{y_j}{d_j}, \quad j = 1, \ldots, J.$$

Then, model (3.1) - (3.11) can be reformulated as follows.

$$
\begin{aligned}
\min \quad & \frac{1}{J} \sum_{j=1}^{J} w_j \\
\text{s. t.} \quad & \mathbf{1}'x \leq C, \\
& l \leq x \leq u, \\
& y = A'x, \\
& y_j \geq \rho d_j, \ \ j = 1, \ldots, J, \\
& y_j \geq C_j, \ \ j = 1, \ldots, J, \\
& w_j \geq \frac{y_j}{d_j} - \frac{1}{J} \sum_{k=1}^{J} \frac{y_k}{d_k}, \ \ j = 1, \ldots, J, \\
& w_j \geq \frac{1}{J} \sum_{k=1}^{J} \frac{y_k}{d_k} - \frac{y_j}{d_j}, \ \ j = 1, \ldots, J, \\
& z_1 = z_2 = z_3 = 1, \\
& \sum_{i=4}^{S+3} z_i \leq \gamma, \\
& x_i \leq M_0 z_i, \ \ \forall \, i = 4, \ldots, S + 3, \\
& z_i \leq x_i, \ \ \forall \, i = 4, \ldots, S + 3, \\
& x_i \in Z^+, \ z_i \in \{0,1\}, \ i = 1, \ldots, S + 3, \ y_j \in Z^+, \ w_j \in \Re, \ j = 1, \ldots, J.
\end{aligned}
\tag{3.12}
$$

Note that problem (3.12) is a mixed integer programming (MIP) with a linear objective function and a system of linear inequality or equality constraints, which can be solved efficiently using available software packages such as AIMMS, CPLEX, and MOSEK.

4 | Numerical Experiment

To illustrate the proposed optimization approach, we have carried out numerical tests on the MIP reformulation using patient administrative data of an emergency department of Singapore. In this section, we report some preliminary numerical results. The tests are

carried out by implementing codes in CPLEX 12.4 installed in a PC with Windows XP Operating System.

4.1 Data

In numerical experiment, we review a total of 41,231 patients of 3-month patient administrative data with approximately 448.2 daily patient visits. Our analysis is conducted based on half-hourly intervals of a 24-hour day. Patient changes to touch points in ED are captured when staff updates patients' locations in the ED information system. To estimate the average touch points on each time interval, we examine patient flow pathways by identifying sequences of patients' touch points from triage to exit. At each time interval, we calculate the average total touch points at the key areas in ED. By assigning different weights to areas, we derive the demand vector d, each d_i, $i = 1, \ldots, 48$, denoting the adjusted demand during the i-th time interval. That is,

$$
\begin{aligned}
d \quad = \quad & (35.35 \ \ 31.09 \ \ 26.37 \ \ 24.57 \ \ 21.19 \ \ 18.86 \ \ 18.82 \ \ 17.6 \ \ 15.75 \ \ 14.88 \ \ 13.29 \ \ 14 \ \ 14.24 \\
& 16.71 \ \ 21.43 \ \ 23.29 \ \ 34.59 \ \ 37.97 \ \ 45.12 \ \ 50.98 \ \ 56.93 \ \ 58.79 \ \ 59.99 \ \ 60.78 \ \ 60.23 \ \ 58.33 \\
& 60.36 \ \ 60.06 \ \ 59.71 \ \ 58.13 \ \ 58.81 \ \ 52.89 \ \ 49.58 \ \ 49.81 \ \ 47.24 \ \ 44.34 \ \ 41.11 \ \ 40.92 \ \ 40.77 \\
& 44.2 \ \ 48.01 \ \ 48.79 \ \ 46.64 \ \ 46.84 \ \ 47.37 \ \ 44.72 \ \ 41.07 \ \ 38.68).
\end{aligned}
$$

From current nursing staff planning and patient demand, we see that the ratio of nursing staff to the adjusted demand varies remarkably over the time intervals of a day. In particular, the work of nursing staff is more intensive during the noon period (10:00 to 14:30) and least busy during midnight period (02:00 to 06:30). This is basically due to the nature of high patient demand during the noon time and low emergency arrivals at midnight hours. The minimum ratio ρ of nursing staff to touch points over the 48 time intervals is calculated as 0.309. This implies that an individual nurse may need to care about three patients during the extreme busy period of a day at some locations in ED, say observation rooms. We also calculate the current mean deviation of the workload ratios as 0.194. This value is subsequently used as a baseline to evaluate the performance of capacity planning rule derived by the MIP model.

4.2 Results

In the test, the nursing staff capacity is set as 60, i.e., $C = 60$. We consider factors concerning shift capacity planning from practical perspective, such as, capacity requirements on manpower supply in each time interval or in each shift, shift times, and total number of minor shifts, and so on. We evaluate various shift capacity planning rules under different considerations. As shown in Table 1, 9 different scenarios are considered in numerical experiment.

In computation, we set $M_0 = 100$, $l_i = 0$ and $u_i = 50$ for $i = 1, \ldots, 51$. In addition, we set $C_j = 0$, $j = 1, \ldots, 48$, if there is no minimum capacity requirement on time intervals. Further, we set $\gamma = 50$ for Case 1. For each scenario under consideration, we solve the corresponding MIP model and report the results in Tables 2-3, where "AM" and "PM" refer to the respective morning and afternoon major shifts. The former table illustrates the respective capacity of nursing staff for the shifts and mean deviations in the above 9 cases. We report the reduction rate of mean deviation for each scenario as shown in Table 2. Specific shift times of both major and minor shifts are elaborated in Table 3.

ED managers can assess the feasibility of capacity planning solutions according to the respective constraints together with other possible considerations in practice. For instance,

Scenario	max # of minor shifts	min # of nurses over time intervals	starting time of minor shifts
Case 1	not required	not required	not required
Case 2	3	not required	not required
Case 3	2	not required	not required
Case 4	1	not required	not required
Case 5	2	not required	one shift starts at 10:00
Case 6	2	not required	one shift starts at 10:30
Case 7	2	not required	one shift starts at 11:00
Case 8	2	not required	one shift starts at 11:30
Case 9	3	13	one shift starts at 10:30

Table 1: Description of Different Scenarios

# of nurses by shift	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
AM shift	13	13	13	21	20	20	21	21	14
PM shift	13	13	13	13	13	13	13	13	10
Night shift	13	13	13	13	13	13	13	13	13
Minor shifts	2	5	12	13	3	2	1	1	10
	2	7	9		11	12	12	12	8
	2	9	9						5
	3								
	2								
	1								
	1								
	6								
	2								
Number of minor shifts	9	3	2	1	2	2	2	2	3
Mean deviation	0.105	0.109	0.113	0.123	0.120	0.122	0.123	0.124	0.104
Mean deviation reduction	45.9%	43.8%	41.8%	36.6%	38.1%	37.1%	36.6%	36.1%	46.4%

Table 2: Summary of Shift Capacity Planning Rules

Case 1 considers the situation with a full freedom in planning, resulting in the least optimization model. The results show that there are 9 minor shifts, in which 3 shifts start in the morning session and the rest starts in the afternoon. The mean deviation reduction is remarkable with a rate of 45.9%, just next to the highest one of Case 9. However this planning solution is impractical in operations. There are too many minor shifts, in particular, during the period of 15:30 to 18:00, a minor shift to be operated in every half-hour interval. In fact, this staffing plan not only increases the complexity of human resource management, but also affects quality of direct care to patients due to high variability of nursing staff during patient duration in ED. Then, we add the constraint on maximum number of minor shifts in Cases 2-9, where γ is chosen from $\{1, 2, 3\}$. The shift time is important in nurse staffing and scheduling involving a variety of practical factors. We thus add some specific requirement on the starting time of minor shift in the model, such as 10:00, 10:30, and so on. In addition, for each time interval of a day, there might be a minimum capacity requirement in order to meet necessary care needs of uncertain arrivals. We illustrate this situation in Case 9. Overall, the solution derived in Case 9, $x = (14, 10, 13, 10, 8, 5)$, appears to be the most promising rule for shift capacity planning. In particular, their shift times concerning minor shifts seem ideal and the reduction rate of mean deviation of the workload is considerably

high as 46%, comparing with other reduction rates under consideration. Decision makers can use the information as a reference in their shift capacity planning.

Shift	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
AM	07:00-15:30	07:00-15:30	07:00-15:30	07:00-15:30	07:00-15:30	07:00-15:30	07:00-15:30	07:00-15:30	07:00-15:30
PM	13:00-21:30	13:00-21:30	13:00-21:30	13:00-21:30	13:00-21:30	13:00-21:30	13:00-21:30	13:00-21:30	13:00-21:30
Night	21:00-07:30	21:00-07:30	21:00-07:30	21:00-07:30	21:00-07:30	21:00-07:30	21:00-07:30	21:00-07:30	21:00-07:30
Minor	10:00-18:30	15:30-24:00	16:00-00:30	15:30-24:00	10:00-18:30	10:30-19:00	11:00-19:30	11:30-20:00	09:00-17:30
	15:30-24:00	16:30-01:00	08:00-16:30		15:30-24:00	15:30-24:00	15:30-24:00	15:30-24:00	15:30-24:00
	16:00-00:30	08:30-17:00							17:00-01:30
	16:30-01:00								
	17:00-01:30								
	17:30-02:00								
	18:00-02:30								
	08:00-16:30								
	08:30-17:00								

Table 3: Shift Times of Both Major and Minor Shifts

In the above discussion, we use the mean deviation of workloads over half-hourly time intervals of a day as a measure to determine the optimal capacity planning rules. For each obtained shift capacity planning scenario, we further analyze the mean deviation of workloads over the corresponding major and minor shifts. In this case, the adjusted demand and the number of nurses on duty by shift are then calculated based on the working hours of specific shifts. The average workload for each shift and its mean deviation among the various shifts for each scenario are reported in Table 4. The results show that current mean workload is higher than that of planning scenarios derived by the model. On top of Case 1, the mean deviation of the workloads over the shift in Case 9 is the smallest. As to the three major shifts, we also find that for all cases under consideration the workload of morning shift is always the highest while that of night shift is the lowest, the workload in the afternoon shift is between these two. In general, this is due to the necessity to maintain certain nursing staff at various areas at ED even if the average emergency arrivals are very low. However, during the working hours of morning shift, the number of patients is high but the shift capacity keeps relatively steady. During the working hours of afternoon shift, there are more extra manpower supply contributed by minor shifts while the number of emergency visits decreases during this period. Hence, the minimum workload among these major shifts (i.e., Row 2-4 of Table 4) can be used as another consideration in decision making.

5 Conclusion

To improve resource utilization at ED, this paper addressed shift scheduling and capacity planning for nursing staff using a mathematical optimization approach. A mixed integer programming model was developed with the objective of minimizing mean deviation of overall nurse workloads over the time of a day. The model provides an optimal shift scheduling rule on the number of total shifts, the shift times and the number of nursing staff by shift. With actual ED patient administrative data, we obtained optimal nurse staffing rules under various scenarios. Hospital managers may use the results for human resource planning appropriately. In the future, we like to apply the results to help plan nurse roster at ED. We are also interested to explore shift capacity planning under the situation of uncertain emergency arrivals using stochastic programming approach.

Shift	Current	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
Morning	2.59	2.16	2.26	2.19	2.24	2.14	2.22	2.19	2.20	2.20
Afternoon	2.04	1.87	1.79	1.83	1.77	1.78	1.76	1.78	1.77	1.87
Night	1.51	1.54	1.58	1.61	1.66	1.69	1.68	1.68	1.68	1.55
Minor	2.47	2.19	1.82	1.80	1.77	2.10	2.10	2.06	1.99	2.11
	2.18	1.93	1.81	2.23		1.84	1.78	1.80	1.80	1.87
	2.19	1.91	2.22							1.89
		1.87								
		1.85								
		1.83								
		1.83								
		2.18								
		2.22								
Mean	2.16	1.95	1.91	1.93	1.86	1.91	1.91	1.90	1.89	1.92
Mean deviation	0.259	0.159	0.218	0.222	0.190	0.168	0.202	0.178	0.166	0.160

Table 4: Average Workload and Mean Deviation over Shifts

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