



STOCHASTIC MODEL AND SYSTEM OPTIMIZATION FOR A CHANNEL AGGREGATION STRATEGY IN COGNITIVE RADIO NETWORKS*

Shunfu Jin, Wuyi Yue and Gang Li

Abstract: In order to improve the service rate of primary user (PU) packets and eliminate the forced termination of secondary user (SU) packets, we propose in this paper a novel channel aggregation strategy in cognitive radio networks. In this strategy, all the channels in a spectrum are aggregated for the transmission of one PU packet, while the transmission of each SU packet occupies only one of the channels in the spectrum. Moreover, by considering that PUs have a higher priority than SUs, a retrial buffer is set for the interrupted SU packets to eliminate any forced termination. We present a queueing model with multiple servers, a retrial buffer and synchronous transmission interruptions. This is done by focusing on a spectrum consisting of multiple channels whereby we can describe the stochastic behavior of the system. Accordingly, in the analysis, we construct a two-dimensional Markov chain and obtain the performance measures in terms of the blocking rates for both types of user packets, the average latency of SU packets, the channel utilization and the system cost function. Numerical results are provided to quantitatively investigate the system performance and optimize the channel aggregation intensity. Furthermore, the Nash equilibrium and the social optimization of the proposed strategy are compared in some cases with a pricing policy for SU packets so that the maximum social profit may be determined.

Key words: *Cognitive radio networks, channel aggregation, performance evaluation, stochastic optimization, Markov chain, Nash equilibrium, pricing policy.*

Mathematics Subject Classification: *68M20, 49K35.*

1 Introduction

With the development of the Internet of Things and the emergence of Big Data, the number and scale of wireless communication businesses are growing dramatically. The demand for wireless spectrum is presenting a huge challenge to the network resources of these businesses. In conventional static spectrum allocation strategy, the spectrum usage is unbalanced and the spectrum utilization is low. The use of cognitive radio networks based on cognitive radio technology is being used to try and solve the problem of spectrum inefficiency [1]. Scholars are currently concentrating their research attention to dynamic spectrum allocation strategies in cognitive radio networks to maximize the social profit of systems and businesses.

One key technique used in dynamic spectrum allocation strategies is channel aggregation. [5] was one to propose a channel aggregation scheme in cognitive radio networks. By assembling several channels together for secondary users (SUs), Lei was able to achieve

*This work was supported in part by National Natural Science Foundation (No. 61472342), China and was supported in part by MEXT, Japan.

higher bandwidth utilization. [6], with the aim of enhancing the throughput, fairness and latency performance, proposed two carrier schedulers with both joint and disjoint queues for channel aggregation in an LTE-advanced system. In [8], in order to obtain a higher service rate, SUs were enabled to sense multiple channels simultaneously, and several idle and consecutive channels were aggregated using a channel bonding technique.

[2] investigated a spectrum assignment method in cognitive Ad-hoc networks. With the help of discontinuous channel access, small channel fragments could be aggregated and further utilized, which dramatically improved the channel utilization.

In some research models, the interrupted SUs were supposed to claim another idle channel rather than directly give up their transmissions. This gave rise to a new type of handoff, namely, the spectrum handoff in cognitive radio networks, namely, the spectrum handoff.

[12], aiming to choose an appropriate channel for each spectrum handoff and resume the unfinished transmission, performed on-demand manner spectrum sensing. They also investigated the influence of the spectrum handoff on channel utilization.

[10] analyzed a proactive handoff approach and proposed a spectrum handoff model with prediction to optimize the spectrum handoff scheme. Chuan found that although a spectrum handoff may increase the channel utilization, a handoff delay and an extra energy cost would be inadvertently introduced. Additionally, if no idle channel was available, the interrupted SUs would be dropped from the system.

Many researchers have also been concentrating on the problem of reducing the forced termination rate. The use of channel reservation has been proposed as one way to address this problem. [11] designed an analytical framework of cognitive radio networks with channel reservation for primary users (PUs), and derived a trade-off between different system parameters. [7] used a channel reservation scheme to deal with the spectrum handoff and employed a fuzzy logic to detect spectrum channel priority. However, use of a channel reservation scheme comes at the cost of a smaller system throughput and a lower channel utilization.

Another way to reduce the forced termination rate is to set a buffer for SUs. [13] presented a framework for admission control by taking into account a finite buffer for interrupted SUs together with newly arriving SUs. The system performance was evaluated by considering that the SUs queueing at the buffer were able to leave the system when they became impatient. In [9], the authors focused on the handoff delay of SUs, then set a finite buffer for newly arriving and interrupted SUs. They also investigated the scheme by using a continuous-time Markov chain. [14] proposed a channel access strategy with α -retry policy. By using a re-trial policy, the forced termination rate could be reduced. Based on the research mentioned above, we can draw a conclusion that the buffer schemes work well for decreasing the forced termination rate. However, it is vital that the buffer size and schedule for the handoff of SUs are set correctly.

In this paper, we propose a new channel aggregation strategy in which all the channels in a spectrum are aggregated as one channel for the transmission of a PU packet, while each SU packet occupies only one of the channels in the spectrum for its transmission. Considering the stochastic behavior of SU packets, we build a kind of discrete-time preemptive re-trial queueing model with multiple servers, a re-trial buffer and synchronous transmission interruptions. Accordingly, we evaluate the system performance under the proposed strategy and optimize the arrival rate of SU packets socially.

The rest of this paper is organized as follows. Section 2 describes the system model for the proposed channel aggregation strategy. In Section 3, an analysis of the model is carried out. Then, the formulas for the performance measures, such as the blocking rate, average delay of SU packets, channel utilization and system cost function are obtained. In Section

4, numerical results are provided to investigate the system performance and optimize the channel aggregation intensity. In Section 5, an appropriate pricing policy is presented to maximize the social profit. Finally, conclusions are drawn and possible future works are suggested in Section 6.

2 System Model

In this paper, establishing that all spectrums in the system to have the same probability nature, we focus on one spectrum, called a “tagged spectrum”, and propose a novel channel aggregation strategy on the tagged spectrum. In order to ensure the transmission quality of PU packets, all the channels in the tagged spectrum can be aggregated together in one channel, called “PU’s channel”, for the transmission of a PU packet. However, the transmission of an SU packet will occupy only one of the channels in the tagged spectrum, so multiple SU packets can be transmitted concurrently if there are multiple idle channels in the spectrum at the same time. Here we call the number of the channels in the tagged spectrum the aggregation intensity. This is denoted by c , which is one system parameter to be optimized.

The PU packets have preemptive priority to occupy the PU’s channel. SU packets, however, can only make opportunistic use of the channels. When a PU packet appears, the transmissions of all the SU packets occupying the channels in the tagged spectrum will be interrupted. All the interrupted SU packets will enter the retrial buffer to protect them from any forced termination.

When an SU packet arrives at the system, if there is no idle channel available in the tagged spectrum, the SU packet will be blocked. Generally speaking, from the view point of user preference, a forced termination is less acceptable than the blocking of a new transmission request. This is because the new transmission request might earn a chance to be switched to other spectrums and receive prompt transmission service. Obviously, the handoff overhead for a new transmission is lighter than that for an interrupted transmission [12].

For this reason, we should consider that the SU packets already in the retrial buffer are supposed to have a higher priority to access the channels in the tagged spectrum over the newly arriving SU packets. Once the tagged spectrum is no longer occupied by a PU packet, the SU packets in the retrial buffer will access the channels in the tagged spectrum immediately to resume their transmissions. Importantly, we note that an SU packet already in the system will never drop away, which eliminates any forced termination of SU packets.

The proposed channel aggregation strategy is demonstrated in Fig. 1.

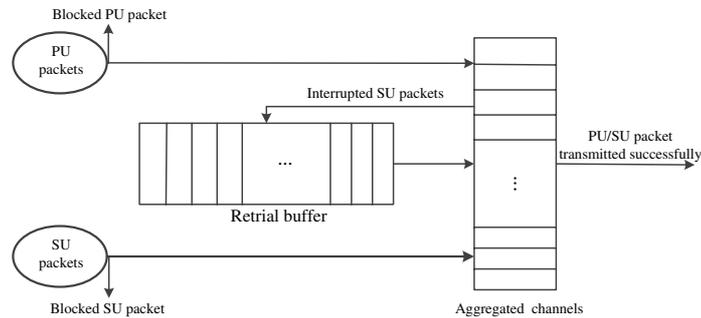


Figure 1: A novel channel aggregation strategy.

From Fig. 1, we describe the working principle of the channel aggregation strategy as follows:

- (1) We assume that SUs sense the channel perfectly. If there is no PU packet in the tagged spectrum, whether or not the channels are occupied by any SU packets, a newly arriving PU packet will occupy the PU's channel. Otherwise, this PU packet will be blocked by the tagged spectrum.
- (2) When an SU packet emerges, if there is at least one idle channel in the tagged spectrum, the central controller will allocate one of the available channels in the spectrum to this SU packet. Otherwise, this SU packet will be blocked by the tagged spectrum.
- (3) If part or all of the channels in the tagged spectrum are being occupied by some SU packets, a newly arriving PU packet will interrupt the transmissions of these SU packets, and occupy the PU's channel with preemptive priority. In this case, all the interrupted SU packets will return to the buffer and wait for a retrial. That is to say, once an SU packet accesses a channel in the tagged spectrum, its transmission will be guaranteed. This results in an improvement in the level of satisfaction with the quality of transmission of SU packets.
- (4) Once the transmission of the PU packet occupying the PU's channel is completed and there are no new arrivals of PU packets, the SU packets in the retrial buffer will access the channels in the tagged spectrum and resume their transmissions. In order to simplify the analysis of this system model, in this paper, we omit the procedure for accumulating the transmission information related to those packets that have been forcibly terminated.
- (5) Since the number of interrupted SU packets at one time is never more than the aggregation intensity c , i.e., the number of channels in the tagged spectrum, we set the retrial buffer size as the aggregation intensity. We note that if the buffer size is less than the aggregation intensity, the forced termination of SU packets can not be eliminated.

We regard the PU packets and SU packets as two types of customers: the PU's channel as one server for PU packets, and each channel in the tagged spectrum as one server for SU packets. From the perspective of the SU packets, we can model the system as a preemptive retrial queue with multiple servers, a retrial buffer and synchronous transmission interruptions.

The time axis is slotted. We consider an early arrival system, namely, the packets are assumed to arrive at the system immediately before the beginning instant n^+ of the n th slot ($n = 1, 2, 3, \dots$), and depart from the system immediately after the end instant n^- of the n th slot ($n = 2, 3, 4, \dots$).

The arriving intervals and transmission times of the packets are supposed to be independent and identically distributed (i.i.d) random variables. The inter-arrival times of the SU packets and PU packets are assumed to follow a geometric distribution with parameters λ_1 ($0 \leq \lambda_1 \leq 1$, $\bar{\lambda}_1 = 1 - \lambda_1$) and λ_2 ($0 \leq \lambda_2 \leq 1$, $\bar{\lambda}_2 = 1 - \lambda_2$), respectively. The transmission time of an SU packet is assumed to follow a geometric distribution with parameter μ_1 ($0 \leq \mu_1 \leq 1$, $\bar{\mu}_1 = 1 - \mu_1$). The transmission time of a PU packet is assumed to follow a geometric distribution with parameter μ_2 ($0 \leq \mu_2 \leq 1$, $\bar{\mu}_2 = 1 - \mu_2$). In the system model considered in this paper, the service rate is in fact the probability that a packet is being completely transmitted in a slot, so the service rate can not be greater than 1. As a result, the time length of a slot should be set appropriately short. For PU packets under the

proposed strategy, the service rate μ_2 on the PU's channel is approximately the sum of service rates for all the channels in the tagged spectrum. Therefore, we have $\mu_2 = \min\{c\mu_0, 1\}$, where μ_0 is the relative service rate of one channel, and c is the aggregation intensity, i.e., the number of channels in the spectrum.

Let $L_n^{(1)} = i$ ($i = 0, 1, 2, \dots, c$) be the number of SU packets in the system at the instant $t = n^+$, and $L_n^{(2)} = j$ ($j = 0, 1$) be the number of PU packets in the system at the instant $t = n^+$. $\{L_n^{(1)}, L_n^{(2)}\}$ constitutes a two-dimensional Markov chain. The state space of this Markov chain is given as follows:

$$\Omega = \{(i, j) : i \in \{0, 1, 2, \dots, c\}, j = 0, 1\}$$

where $(0, 0)$ denotes that there are no packets in the system; $(i, 0)$ ($1 \leq i \leq c$) denotes that there are i SU packets occupying i channels in the tagged spectrum and no PU packets in the system; $(i, 1)$ denotes that there is a PU packet occupying the PU's channel and i SU packets in the retrieval buffer.

3 Performance Analysis

3.1 Stationary Probability Distribution

We define the system phase as the total number of SU packets in the system. Let \mathbf{P} be the state transition probability matrix for the system phases. According to different system phases, \mathbf{P} can be given as a $(c + 1) \times (c + 1)$ block-structured matrix as follows:

$$\mathbf{P} = \begin{bmatrix} \mathbf{A}_{0,0} & \mathbf{A}_{0,1} & & & \\ \mathbf{A}_{1,0} & \mathbf{A}_{1,1} & \mathbf{A}_{1,2} & & \\ \vdots & \vdots & \vdots & \ddots & \\ \mathbf{A}_{c-1,0} & \mathbf{A}_{c-1,1} & \mathbf{A}_{c-1,2} & \dots & \mathbf{A}_{c-1,c} \\ \mathbf{A}_{c,0} & \mathbf{A}_{c,1} & \mathbf{A}_{c,2} & \dots & \mathbf{A}_{c,c} \end{bmatrix} \quad (3.1)$$

where the sub-matrix $\mathbf{A}_{u,v}$ is the transition probability matrix from the system phase u ($u = 0, 1, \dots, c$) to the system phase v ($v = 0, 1, \dots, c$). Considering that no buffer is prepared for PU packets, there is at most one PU packet, namely $j = 0$ or $j = 1$, in the system. It is easy to find that each sub-matrix $\mathbf{A}_{u,v}$ has an order of 2×2 structure. $\mathbf{A}_{u,v}$ is discussed as follows.

- (1) At the instant $t = n^+$, the system phase is $u = 0$, i.e., there are no SU packets in the system, and the system phase will be v ($v = 0, 1$) at the instant $t = (n + 1)^+$.

If the system phase $v = 0$, namely, there are also no packets in the system at the instant $t = (n + 1)^+$, the transition probability matrix $\mathbf{A}_{0,0}$ is given as follows:

$$\mathbf{A}_{0,0} = \begin{bmatrix} \bar{\lambda}_1 \bar{\lambda}_2 & \lambda_2 \\ \bar{\lambda}_1 \bar{\lambda}_2 \mu_2 & \lambda_2 \mu_2 + \bar{\mu}_2 \end{bmatrix}, \quad v = 0.$$

If the system phase $v = 1$, namely, there is an SU packet in the system at the instant $t = (n + 1)^+$, the transition probability matrix $\mathbf{A}_{0,1}$ is given as follows:

$$\mathbf{A}_{0,1} = \begin{bmatrix} \lambda_1 \bar{\lambda}_2 & 0 \\ \lambda_1 \bar{\lambda}_2 \mu_2 & 0 \end{bmatrix}, \quad v = 1.$$

- (2) At the instant $t = n^+$, the system phase is u ($u = 1, 2, \dots, c - 1$), the system phase will be v ($v = 0, 1, \dots, u + 1$) at the instant $t = (n + 1)^+$.

The system phase $v = 0$ means that there are no SU packets in the system at the instant $t = (n + 1)^+$. In this case, all the SU packets in the system complete their transmissions and leave the system together. At the same time, there are no new SU packet arrivals at the system. So the transition probability matrix $\mathbf{A}_{u,0}$ is given as follows:

$$\mathbf{A}_{u,0} = \begin{bmatrix} \bar{\lambda}_1 \bar{\lambda}_2 \mu_1^u & \lambda_2 \mu_1^u \\ 0 & 0 \end{bmatrix}, \quad v = 0. \tag{3.2}$$

The system phase $v > 0$ means that there are v ($v \geq 1$) SU packets in the system at the instant $t = (n + 1)^+$. One case is that $u \geq v$ and $(u - v)$ of the SU packets in the system complete their transmissions and leave the system together. At the same time, no new SU packets arrive at the system. A second case is that $u \geq v$ and $(u - v + 1)$ of the SU packets in the system complete their transmissions and leave the system. Meanwhile, a new SU packet arrives at the system. A third case ($v = u + 1$) means that all the SU packets in the system do not complete their transmissions and a new SU packet arrives at the system. So the transition probability matrix $\mathbf{A}_{u,v}$ is given as follows:

$$\mathbf{A}_{u,v} = \begin{cases} \begin{bmatrix} \bar{\lambda}_2 \left(\bar{\lambda}_1 \binom{u}{v} \mu_1^{u-v} \bar{\mu}_1^v + \lambda_1 \binom{u}{v-1} \mu_1^{u-v+1} \bar{\mu}_1^{v-1} \right) & \lambda_2 \binom{u}{v} \mu_1^{u-v} \bar{\mu}_1^v \\ 0 & 0 \end{bmatrix}, & 1 \leq v < u \\ \begin{bmatrix} \bar{\lambda}_2 \left(\bar{\lambda}_1 \bar{\mu}_1^v + \lambda_1 \binom{u}{1} \mu_1 \bar{\mu}_1^{u-1} \right) & \lambda_2 \bar{\mu}_1^v \\ \bar{\lambda}_1 \bar{\lambda}_2 \mu_2 & \lambda_2 \mu_2 + \bar{\mu}_2 \end{bmatrix}, & v = u \\ \begin{bmatrix} \lambda_1 \bar{\lambda}_2 \bar{\mu}_1^u & 0 \\ \lambda_1 \bar{\lambda}_2 \mu_2 & 0 \end{bmatrix}, & v = u + 1. \end{cases} \tag{3.3}$$

- (3) At the instant $t = n^+$, the system phase is $u = c$, i.e., there are u SU packets in the system, and the system phase will be v ($v = 0, 1, \dots, c$) at the instant $t = (n + 1)^+$. Similar to the matrix structures shown in Eqs. (3.2)-(3.3), the transition probability matrix $\mathbf{A}_{u,v}$ ($v = 0, 1, \dots, c - 1$) can be given as follows:

$$\mathbf{A}_{c,0} = \begin{bmatrix} \bar{\lambda}_1 \bar{\lambda}_2 \mu_1^c & \lambda_2 \mu_1^c \\ 0 & 0 \end{bmatrix}, \quad v = 0,$$

$$\mathbf{A}_{c,v} = \begin{bmatrix} \bar{\lambda}_2 \left(\bar{\lambda}_1 \binom{c}{v} \mu_1^{c-v} \bar{\mu}_1^v + \lambda_1 \binom{c}{v-1} \mu_1^{c-v+1} \bar{\mu}_1^{v-1} \right) & \lambda_2 \binom{c}{v} \mu_1^{c-v} \bar{\mu}_1^v \\ 0 & 0 \end{bmatrix}, \quad 1 \leq v \leq c-1.$$

The system phase $v = c$ means that there are c SU packets in the system at the instant $t = (n + 1)^+$. In this phase, one possible case is that one of the SU packets leaves the system. The other possible case is that a new SU packet arrives at the system. So the transition probability matrix $\mathbf{A}_{c,c}$ can be given as follows:

$$\mathbf{A}_{c,c} = \begin{bmatrix} \bar{\lambda}_2 \left(\bar{\mu}_1^c + \lambda_1 \binom{c}{1} \mu_1 \bar{\mu}_1^{c-1} \right) & \lambda_2 \bar{\mu}_1^c \\ \bar{\lambda}_1 \bar{\lambda}_2 \mu_2 & \lambda_2 \mu_2 + \bar{\mu}_2 \end{bmatrix}, \quad v = c.$$

Now, all the sub-matrixes in \mathbf{P} have been presented.

The structure of the transition probability matrix \mathbf{P} indicates that the two-dimensional Markov chain $\{L_n, L_n^{(1)}\}$ is non-periodic, irreducible and positive recurrent. Letting $\pi_{i,j}$ be the steady-state distribution of the two-dimensional Markov chain, $\pi_{i,j}$ can be given as follows:

$$\pi_{i,j} = \lim_{n \rightarrow \infty} P\{L_n = i, L_n^{(1)} = j\}. \tag{3.4}$$

Let $\mathbf{\Pi}_i$ be the steady-state probability vector for the system being at phase i . $\mathbf{\Pi}_i$ can be given as follows:

$$\mathbf{\Pi}_i = (\pi_{i,0}, \pi_{i,1}), \quad 0 \leq i \leq c. \tag{3.5}$$

Combining the balance equation and normalization condition for the Markov chain mentioned above, we have

$$\begin{cases} (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \dots, \mathbf{\Pi}_c) \mathbf{P} = (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \dots, \mathbf{\Pi}_c) \\ (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \dots, \mathbf{\Pi}_c) \mathbf{e} = 1 \end{cases} \tag{3.6}$$

where \mathbf{e} is a one's column vector.

Eq. (3.6) is a linear system of equations with $2 \times (c + 1)$ unknowns. By using a Gaussian elimination method, we establish an iterative algorithm to calculate the steady-state distribution $\mathbf{\Pi} = (\pi_{0,0}, \pi_{0,1}, \dots, \pi_{c,0}, \pi_{c,1})$. The main steps of the iterative algorithm are given in Table 1.

Table 1: Iteration algorithm to calculate $\mathbf{\Pi}$.

Input: State transition probability matrix \mathbf{P} , an arbitrarily small number ε .
Output: Steady-state distribution $\mathbf{\Pi}$.
Begin
(1) Set $\tau = 2 \times (c + 1)$;
(2) Set the maximum error ε ;
(3) Set the initial iterative time as $m = 0$ and the initial value $\mathbf{\Pi}^{(0)}$;
(4) Construct a $\tau \times \tau$ matrix \mathbf{G} by replacing an arbitrary column of matrix $(\mathbf{P} - \mathbf{E})$ with column vector \mathbf{e} . Here \mathbf{E} is a $\tau \times \tau$ unit matrix;
(5) Construct an $1 \times \tau$ row vector $\mathbf{b} = (\mathbf{0}, 1)$, where $\mathbf{0}$ is an $1 \times (\tau - 1)$ zero row vector;
(6) Split matrix \mathbf{G} into three $\tau \times \tau$ matrixes denoted as \mathbf{U} , \mathbf{V} and \mathbf{R} . \mathbf{U} is the strictly lower triangular part of \mathbf{G} , \mathbf{V} is the strictly upper part of \mathbf{G} , and \mathbf{R} is the diagonal part of \mathbf{G} ;
(7) While $\ \mathbf{\Pi}^{(m+1)} - \mathbf{\Pi}^{(m)}\ > \varepsilon$
$m = m + 1$;
$\mathbf{\Pi}^{(m+1)} = -\mathbf{\Pi}^{(m)} \mathbf{V}(\mathbf{R} + \mathbf{U})^{-1} + \mathbf{b}(\mathbf{R} + \mathbf{U})^{-1}$;
Endwhile.
End

By selecting a suitably small ε , we can obtain the steady-state distribution $\mathbf{\Pi}$ with enough precision.

3.2 Performance Measures and Cost Function

We suppose that the transmissions for the PU packets are independent of those for the SU packets. Since there is no buffer for PU packets, the transmission process of PU packets

can be regarded as a simple pure losing queueing model with a single server. Let p_2 be the probability that the tagged spectrum is occupied by a PU packet in the system. p_2 is then given as follows:

$$p_2 = \lim_{n \rightarrow \infty} P \left\{ L_n^{(2)} = 1 \right\} = \frac{\lambda_2}{\lambda_2 + \mu_2}.$$

We define the blocking rate B_{pu} of PU packets as the probability that a newly arriving PU packet is blocked by the system. We note that only when the PU's channel is occupied by a PU packet will the newly arriving PU packets be blocked by the system. The blocking rate B_{pu} of PU packets is then given as follows:

$$B_{pu} = \lambda_2 p_2 \bar{\mu}_2 = \frac{\lambda_2^2 \bar{\mu}_2}{\lambda_2 + \mu_2}. \quad (3.7)$$

In cognitive radio networks, the transmission of an SU packet can be influenced by PU packets. As a result, the performance measures of SU packets are affected by PU packets' activities. This is important to note as we next mathematically derive some required performance measures for the SU packets.

We define the blocking rate B_{su} of SU packets as the probability that a newly arriving SU packet is blocked by the system. When an SU packet arrives at the system, if no channel is available, the newly arriving SU packet will not be allowed by the system.

In an early arrival system, the newly arriving SU packet will be blocked by the system in the following four cases: (a) In the previous slot, a PU packet occupies the PU's channel and this PU packet doesn't complete its transmission at the end of the slot. So, the new SU packet arriving at the current slot will be blocked. (b) When a PU packet and an SU packet arrive at the system simultaneously, the newly arriving SU packet will be blocked due to the higher priority of the PU packets. (c) In the previous slot, all the channels in the tagged spectrum are occupied by SU packets. If none of the SU packets complete their transmission at the end of the slot and there are no new PU packet arrivals at the current slot, the newly arriving SU packet can not access the channel. (d) In the previous slot, there is a PU packet occupying the PU's channel and the retrial buffer is full of interrupted SU packets. If the transmission of the PU packet is completed at the end of the previous slot and there are no PU packet arrivals, the SU packets in the retrial buffer will occupy the vacated channels in the spectrum. So, the newly arriving SU packet has to leave the system. Conclusively, the blocking rate B_{su} of SU packets can be given as follows:

$$\begin{aligned} B_{su} &= \lambda_1(p_2 \bar{\mu}_2 + (p_2 \mu_2 + (1 - p_2)) \lambda_2 + \pi_{c,0} \bar{\lambda}_2 \bar{\mu}_1^c + \pi_{c,1} \bar{\lambda}_2 \mu_2) \\ &= \frac{\lambda_1 \lambda_2 + \lambda_1 \lambda_2^2 \mu_2}{\lambda_2 + \mu_2} + \pi_{c,0} \lambda_1 \bar{\lambda}_2 \bar{\mu}_1^c + \pi_{c,1} \lambda_1 \bar{\lambda}_2 \mu_2. \end{aligned} \quad (3.8)$$

We define the latency of an SU packet as the time duration from the instant at which an SU packet joins the system to the instant that the SU packet is successfully transmitted. Considering an early arrival system, during the transmission period of an SU packet, possible interruption occurs at the beginning instant of every slot other than the first. We suppose that an SU packet will experience k interruptions before it is transmitted successfully. For each interruption, the SU packet has to wait for a period of time, during which several PU packets will perform their transmissions. The latency of an SU packet is the sum of the waiting time and the transmission time. This means the average latency W_{su} of SU packets

can be expressed as follows:

$$\begin{aligned}
 W_{su} &= 1 + \sum_{j=1}^{\infty} \sum_{k=0}^{\infty} k \left(\frac{1}{\mu_2} + 1 \right) \lambda_2^k \bar{\lambda}_2 (j-1) \mu_1 \bar{\mu}_1^{j-1} \\
 &= 1 + \frac{\bar{\mu}_1}{\mu_1} \left(\frac{\lambda_2}{\bar{\lambda}_2 \mu_2} + 1 \right).
 \end{aligned}
 \tag{3.9}$$

We define the channel utilization θ of the system as the probability that the channels in the tagged spectrum are being occupied by PU or SU packets in a slot. According to the working principle of the proposed channel aggregation strategy, the channel states are classified into three categories: (a) With probability $\pi_{0,0}$, there is no packet in the system, i.e., all the channels in the spectrum are idle. For this case, the channel utilization is 0. (b) With probability p_2 , the tagged spectrum is occupied by a PU packet. Because all the channels in the tagged spectrum are aggregated together for the transmission of one PU packet, the channel utilization for this state is up to 100%. (c) With probability $\pi_{i,0}$, there are i ($i = 1, 2, \dots, c$) SU packets occupying part channels in the tagged spectrum. Since the transmission of each SU packet occupies only one of the channels in the tagged spectrum, the channel utilization for this case is $\frac{i}{c}$. In summary, the channel utilization θ can be given as follows:

$$\begin{aligned}
 \theta &= p_2 + \sum_{i=1}^c \frac{i\pi_{i,0}}{c} \\
 &= \frac{\lambda_2}{\lambda_2 + \mu_2} + \sum_{i=1}^c \frac{i\pi_{i,0}}{c}.
 \end{aligned}
 \tag{3.10}$$

We know that with an increase in the aggregation intensity, the blocking rates for the two types of user packets and the average latency of SU packets will decrease. However, the channel utilization will also decrease. This means that in the channel aggregation strategy proposed in this paper there is a trade-off among different performance measures when setting the aggregation intensity. In order to improve the beneficial effects and reduce any negative effects on the system performance, it is necessary to optimize the aggregation intensity. Hence, we construct a system cost function $F(c)$ as follows:

$$F(c) = f_1 B_{pu} + f_2 B_{su} + f_3 W_{su} + \frac{f_4}{\theta} + f_5 c
 \tag{3.11}$$

where f_1, f_2, f_3, f_4 and f_5 are assumed to be the cost impact factors of the blocking rate B_{pu} of PU packets, blocking rate B_{su} of SU packets, average latency W_{su} of SU packets, channel utilization θ and channel aggregation intensity c , respectively.

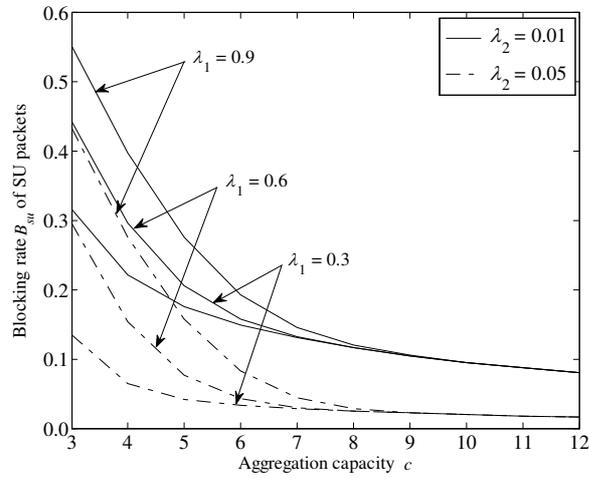
The system cost function can achieve the minimum value when the channel aggregation intensity is optimized. The value of the channel aggregation intensity c minimizing the cost function $F(c)$ is the optimal channel aggregation intensity c^* . Therefore, c^* can be given as follows:

$$c^* = \arg \min\{F(c)\}.
 \tag{3.12}$$

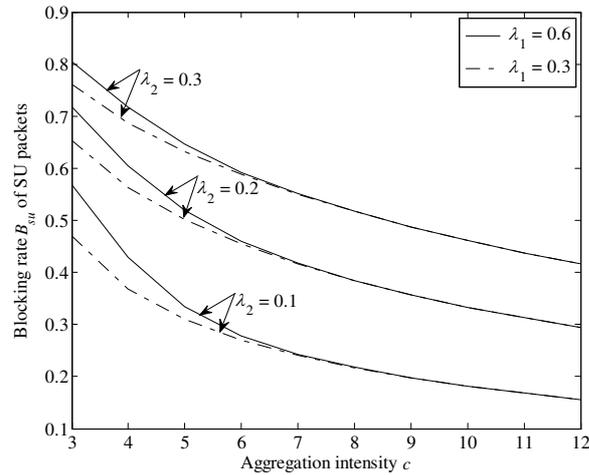
4 Numerical Results

In this section, the system performance for the proposed channel aggregation strategy is investigated using numerical results. In the numerical results, we set the service rate for PU packets as $\mu_2 = c\mu_0$ ($\mu_0 = 0.05$), and the service rate for SU packets as $\mu_1 = 0.2$.

Figure 2 illustrates the blocking rate B_{su} of SU packets versus the channel aggregation intensity c for different arrival rates λ_1 of SU packets and λ_2 of PU packets.



(a) $\lambda_2 = 0.01$ and $\lambda_2 = 0.05$



(b) $\lambda_1 = 0.3$ and $\lambda_1 = 0.6$

Figure 2: Blocking rate B_{su} of SU packets vs. channel aggregation intensity c .

From Fig. 2, we observe that for the same arrival rate λ_1 of PU packets or the same arrival rate λ_2 of SU packets, the blocking rate B_{su} of SU packets will decrease as the channel aggregation intensity c increases. This is because the higher the channel aggregation intensity is, the greater the service rate of PU packets is, the less likely it is that the PU's channel is occupied by a PU packet, and the more likely it is that a newly arriving SU packet is able to access the spectrum. So, the blocking rate of SU packets will be lower.

In addition, from Fig. 2 (a) we find that the arrival rate λ_1 of SU packets has an impact on the blocking rate of SU packets. We see that for the same arrival rate λ_2 of PU packets, such as $\lambda_2 = 0.01$, when the channel aggregation intensity c is lower, the blocking rate B_{su} of SU packets shows a sharp increasing trend as the arrival rate λ_1 of SU packets increases. The reason being is that when the arrival rate of SU packets is higher, the more SU packets

there will be occupying the channels in the tagged spectrum, the greater the likelihood is that all the channels will be occupied, so any newly arriving SU packets are more likely to be blocked. As the channel aggregation intensity c continuously increases, the impact of the arrival rate λ_1 of SU packets on the blocking rate lessens. This is because when the channel aggregation intensity is greater than a certain value, such as $c \geq 9$, all the channels are more likely to be occupied by SU packets. So the blocking rate of SU packets tends to be fixed.

On the other hand, from Fig. 2 (b) we find that for the same arrival rate λ_1 of SU packets, such as $\lambda_1 = 0.3$, and the same channel aggregation intensity c , the higher the arrival rate λ_2 of PU packets is, the greater the blocking rate B_{su} of the SU packets will be. This is because as the arrival rate of PU packets increases, the possibility that the PU's channel will be occupied by a PU packet is higher, so the possibility of a new SU packet being able to access the channel will be lower. As a result, the blocking rate of SU packets will increase.

In addition, we examine the influence of the channel aggregation intensity c on the average latency W_{su} of SU packets for different arrival rates λ_2 of PU packets in Fig. 3.

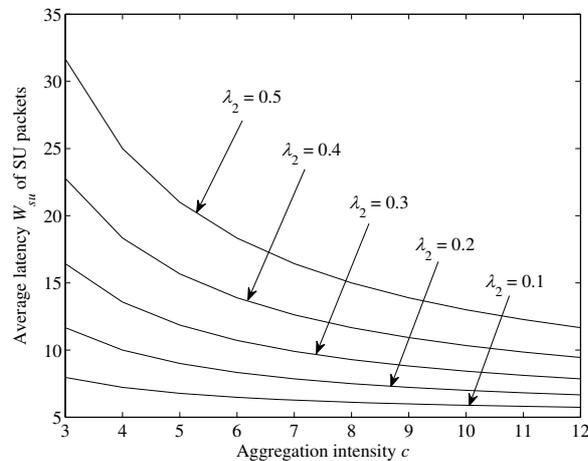


Figure 3: Average latency W_{su} of SU packets vs. channel aggregation intensity c .

In Fig. 3, we observe that for the same arrival rate λ_2 of PU packets, the average latency W_{su} of SU packets will decrease as the channel aggregation intensity c increases. The reason is that the larger the channel aggregation intensity is, the quicker the PU packets will be transmitted. Therefore the waiting time for an SU packet at the system will be shorter, and this will result in a decrease in the average latency of SU packets.

Moreover, from Fig. 3, we find that for the same channel aggregation intensity c , the average latency W_{su} of SU packets will increase as the arrival rate λ_2 of PU packets increases. This is because as the arrival rate of PU packets increases, the possibility that the PU's channel will be occupied by a PU packet increases, so the time period for an SU packet waiting in the buffer will be longer. Therefore, the average latency of SU packets will be greater.

Figure 4 illustrates the channel utilization θ versus the channel aggregation intensity c for different arrival rates λ_1 of SU packets and λ_2 of PU packets.

In Fig. 4, it can be observed that for the same arrival rate λ_1 of SU packets and the same arrival rate λ_2 of PU packets, the channel utilization θ will decrease as the channel aggregation intensity c increases. The reason is that the larger the channel aggregation

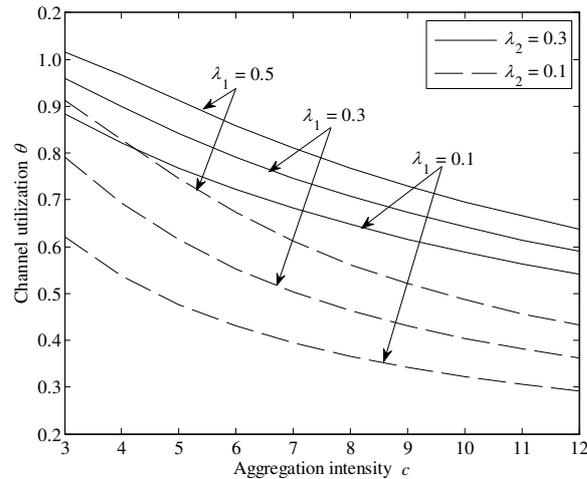


Figure 4: Channel utilization θ vs. channel aggregation intensity c .

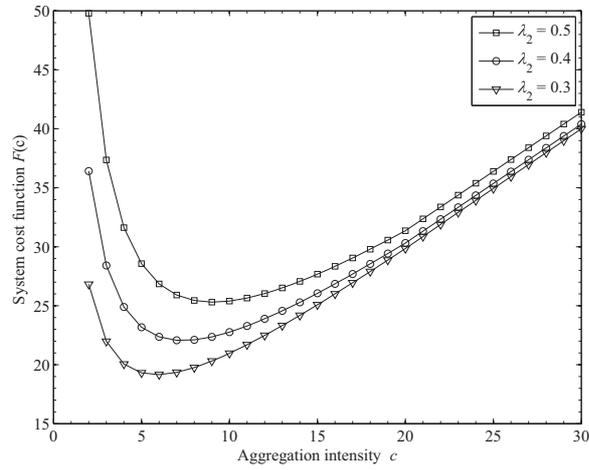
intensity is, the quicker the PU packets are transmitted. For a certain traffic load of SU packets, there is a higher possibility that the channels will be idle. Therefore, the channel utilization will be lower.

When the channel aggregation intensity c is fixed, the channel utilization θ will increase as the arrival rates λ_1 of SU packets or λ_2 of PU packets increase. This is because the higher the arrival rates λ_1 of SU packets or λ_2 of PU packets are, the greater the probability is that the channels in the tagged spectrum will be occupied by PU or SU packets, so the channel utilization will be greater.

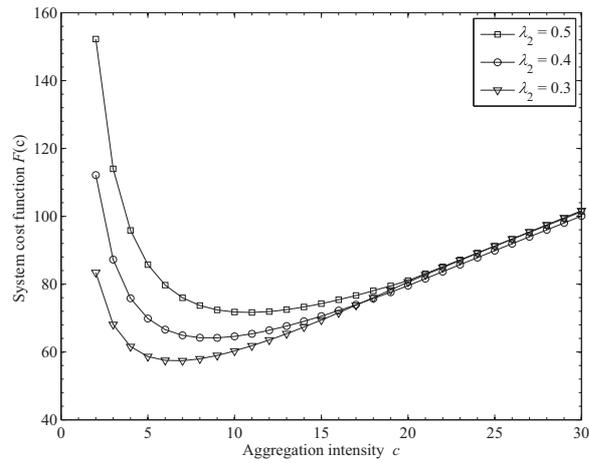
As an example, we set the arrival rate of SU packets as $\lambda_1 = 0.3$, and the service rate on each channel for SU packets as $\mu_1 = 0.2$. Moreover, we consider the service rate on each channel for a PU packet as $\mu_0 = 0.05$. By setting $\lambda_2 = 0.3$, $\lambda_2 = 0.4$ and $\lambda_2 = 0.5$ as an example, we plot how the system cost function $F(c)$ changes with respect to the channel aggregation intensity c for two groups of cost impact factors in Fig. 5. In Fig. 5 (a), we set the cost impact factors as $f_1 = f_2 = f_3 = f_4 = f_5 = 1$; in Fig. 5 (b), the cost impact factors are set as $f_1 = 5$, $f_2 = 2$, $f_3 = 3$, $f_4 = 7$, $f_5 = 2$.

From Fig. 5, we observe that all the cost functions experience two stages. In the first stage, the cost function $F(c)$ will decrease along with an increase in the channel aggregation intensity c . During this stage, the greater the channel aggregation intensity is, the lower the blocking rates for both PU packets and SU packets will be. Moreover, the average latency of SU packets will decrease sharply. We note that the cost introduced by the decrease in the channel utilization changes slowly. So, the system cost illustrates an overall decreasing trend in the first stage. In the second stage, the cost function $F(c)$ will increase with an increase in the channel aggregation intensity c . During this period, the higher the channel aggregation intensity is, the greater effect the channel utilization and the channel aggregation intensity will have on the cost function. Namely, when the channel aggregation intensity exceeds a critical value, the channel utilization and channel aggregation intensity will play more important roles in influencing the system cost function. A smaller channel utilization and a greater channel aggregation intensity will raise the system cost function.

In summary, the cost function $F(c)$ invariably exists as a minimal value $F(c^*)$ for all the system parameters when the channel aggregation intensity c is set to an optimal value



(a) $f_1 = f_2 = f_3 = f_4 = f_5 = 1$



(b) $f_1 = 5, f_2 = 2, f_3 = 3, f_4 = 7, f_5 = 2$

Figure 5: System cost function $F(c)$ vs. channel aggregation intensity c .

c^* . For two groups of cost impact factors, the optimal channel aggregation intensities with different arrival rates of PU packets are shown in Table 2.

Table 2: Optimal channel aggregation intensity c^* .

f_1	f_2	f_3	f_4	f_5	Arrival rate λ_2	Minimal cost $F(c^*)$	Optimal channel aggregation intensity c^*
1	1	1	1	1	0.3	19.17	6
					0.4	22.08	7
					0.5	25.33	9
5	2	3	7	2	0.3	57.41	7
					0.4	64.17	9
					0.5	71.69	11

5 Admission Fee

In this section, we firstly investigate the Nash equilibrium and the socially optimal behaviors of SUs, then we propose an appropriate pricing policy to maximize the social profit.

5.1 Nash Equilibrium Behavior

Every SU is individually selfish and tries to access the system for its own benefit [3]. If an SU packet is admitted to the system, it will be transmitted successfully and get reward R . Conversely, if an SU packet tries to access the system but fails, it will not be rewarded. No matter whether an SU packet is admitted to the system or not, a trial cost T ($T < R$), such as the cost in channel sensing, propagation delay and sojourn time, has to be paid. Therefore, SUs will adjust their transmission requests.

We consider an SU packet's strategy with probability q . Namely, an SU packet decides to join the buffer with a probability of q ($0 \leq q \leq 1$), and leaves the system with probability \bar{q} ($\bar{q} = 1 - q$). Since SU packets are allowed to make their own decisions, there will be a non-cooperative and symmetric game among the SU packets. In the presence of the joining probability of SU packets, the effective arrival rate λ_1^e deviates from the potential arrival rate λ_1 with $\lambda_1^e = q_e \lambda_1$, where q_e is the Nash equilibrium probability. The effective arrival rate λ_1^e in the Nash equilibrium state is called the Nash equilibrium arrival rate.

We define the individual benefit function U_I for one SU packet that tries to join the system as follows:

$$U_I = \left(1 - \frac{B_{su}}{\lambda_1}\right) \times R - T \quad (5.1)$$

where B_{su} is the blocking rate of SU packets given in Eq. (3.8).

With an increase in the arrival rate λ_1 of SU packets, the blocking rate B_{su} of SU packets will increase monotonically. Hence the individual benefit U_I for one SU packet is a decreasing function about the arrival rate λ_1 of SU packets. In other words, as the arrival rate of SU packets increases, the blocking rate of SU packets grows and the individual benefit decreases. Since all SUs are individually selfish, they all try their best to access the system. Provided the benefit is positive, the arrival rate of SU packets will continue to grow. If there is at least one solution for the inequality $U_I \geq 0$ within the closed interval $[\lambda_{min}, \lambda_{max}]$, the maximal value of the solutions is the Nash equilibrium arrival rate λ_1^e . Otherwise, $\lambda_1^e = \lambda_{min}$. No SU

packet has any incentive to deviate unilaterally from the Nash equilibrium arrival rate. We discuss the Nash equilibrium of the proposed strategy as follows:

- (1) Letting $\lambda_1 = \lambda_{min}$, if $\frac{T}{(1 - B_{su}/\lambda_1)} > R$, the individual benefit function U_I for one SU packet is less than zero. For this case, even if all the SU packets arrive at the system at the lowest arrival rate, the individual benefit for one SU packet is negative. Therefore, $\lambda_1^e = \lambda_{min}$ is a Nash equilibrium arrival rate and no other Nash equilibrium arrival rates exist. That is to say, the dominant strategy is one where SU packets arrive at the system at the lowest possible rate.
- (2) Letting $\lambda_1 = \lambda_{max}$, if $\frac{T}{(1 - B_{su}/\lambda_1)} \leq R$, the individual benefit function U_I for one SU packet is no less than zero. For this case, even if all the SU packets arrive at the system at the maximum arrival rate, they all enjoy non-negative individual benefits. Therefore, $\lambda_1^e = \lambda_{max}$ is a Nash equilibrium arrival rate and no other Nash equilibrium arrival rate is possible. That is to say, the dominant strategy is one where SU packets arrive at the system at the highest rate.
- (3) Letting $\lambda_{min} < \lambda_1 < \lambda_{max}$, if $\frac{T}{(1 - B_{su}/\lambda_1)} > R$, some SU packets will suffer negative profits, so this can not be a Nash equilibrium strategy. On the other hand, if $\frac{T}{(1 - B_{su}/\lambda_1)} < R$, some SU packets will receive positive profits, so this also can not be a Nash equilibrium strategy. Therefore, there exists a unique Nash equilibrium strategy $\lambda_{min} < \lambda_1^e < \lambda_{max}$ satisfying $\frac{T}{(1 - B_{su}/\lambda_1)} = R$. In this case, λ_1^e is a mixed Nash equilibrium arrival rate.

We investigate the monotonicity of the individual benefit U_I for one SU packet using numerical experiments. Referencing [4] and [14], we set the parameters as follows: $\lambda_{min} = 0.05$, $\lambda_{max} = 0.50$, $\mu_1 = 0.2$, $\mu_0 = 0.05$, $c = 3$, $R = 20$ and $T = 15$. By setting $\lambda_2 = 0.01$, 0.02 and 0.03 , respectively, we show the change trend of individual benefit U_I for one SU packet versus arrival rate λ_1 of SU packets in Fig. 6.

In Fig. 6, we find that with the parameters set above, all the individual benefits U_I show downward trends as the arrival rate λ_1 of SU packets increases. We also find that all the individual benefits U_I go through $U_I = 0$, i.e., there are always values of λ_1^e subject to $U_I = 0$. That is to say, a Nash equilibrium behavior for our proposed strategy exists.

5.2 Socially Optimal Behavior

In the system design, it is necessary to consider the level of social benefit derived during operation as well as the benefit to the individual users. In this subsection, we turn our attention to the socially optimal behavior of SU packets. We define the social profit function U_S of SU packets as follows:

$$U_S = \lambda_1 * \left(\left(1 - \frac{B_{su}}{\lambda_1} \right) * R - T \right). \tag{5.2}$$

By maximizing the social profit, we can derive the socially optimal arrival rate λ_1^* , where λ_1^* is given as follows:

$$\lambda_1^* = \arg \max_{\lambda_1 \in [\lambda_{min}, \lambda_{max}]} U_S. \tag{5.3}$$

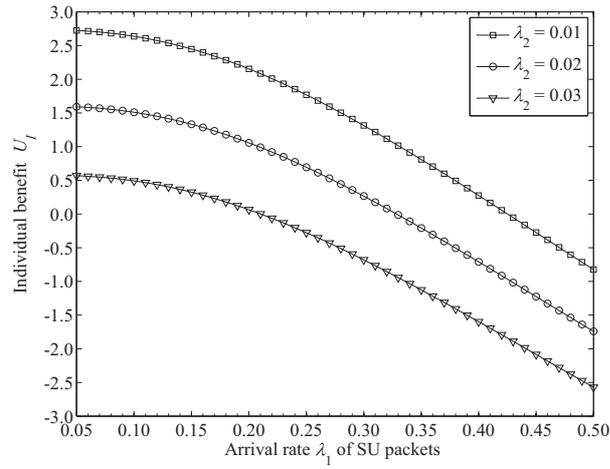


Figure 6: Individual benefit U_I vs. arrival rate λ_1 of SU packets.

With the same parameters as used in Fig. 6, we show how the social profit U_S changes with respect to the arrival rate λ_1 of SU packets in Fig. 7.

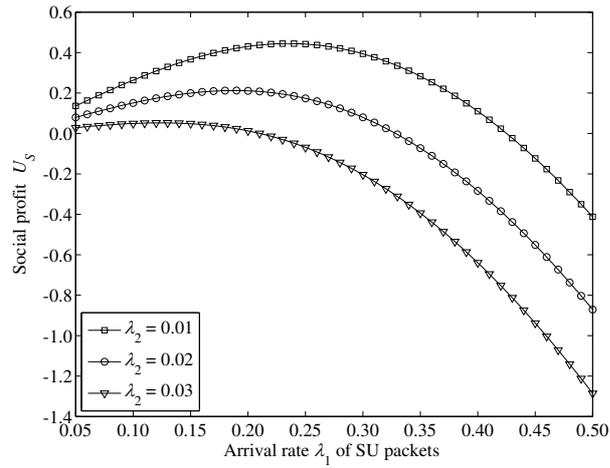


Figure 7: Social profit U_S vs. arrival rate λ_1 of SU packets.

In Fig. 7, we find that there is always a socially optimal arrival rate λ_1^* and a maximal social profit U_S^* for all the arrival rates λ_2 of PU packets.

Combining the results given in Figs. 6 and 7, we summarize the Nash equilibrium arrival rate λ_1^e and the socially optimal arrival rate λ_1^* in Table 3.

From Table 3, we conclude that optimizing the individual benefit leads to a higher arrival rate of SU packets than that socially desired. This issue can be addressed by imposing an appropriate admission fee for SU packets.

Table 3: Numerical results with Nash equilibrium and socially optimal behaviors.

Arrival rate λ_2 of PU packets	Nash equilibrium arrival rate λ_1^e of SU packets	Socially optimal arrival rate λ_1^* of SU packets
0.01	0.428	0.240
0.02	0.330	0.188
0.03	0.215	0.126

5.3 Pricing Policy

One approach that would oblige the SU packets to adopt the socially optimal arrival rate is to charge a fee to the SU packets joining the system. We assume the base station acts as a pricing agent and imposes an admission fee on all the SU packets transmitted successfully. So, for the channel aggregation strategy proposed in this paper, we present a pricing policy. It is worth mentioning that the admission fee f is different from the trial cost T . The admission fee f is only imposed on the SU packets transmitted successfully, whereas the trial fee T is a cost that each arriving SU packet has to pay.

When the pricing policy is implemented, the individual benefit U'_I for one SU packet will be given as follows:

$$U'_I = \left(1 - \frac{B_{su}}{\lambda_1}\right) * (R - f) - T. \tag{5.4}$$

Substituting the arrival rate λ_1 of SU packets in Eq. (5.4) with the socially optimal arrival rate λ_1^* of SU packets given in Table 3 and letting $U'_I = 0$, we can calculate the admission fee f as follows:

$$f = R - \frac{T}{\left(1 - \frac{B_{su}}{\lambda_1}\right)}. \tag{5.5}$$

With the socially optimal arrival rates λ_1^* given in Table 3, we calculate the blocking rate B_{su} using Eq. (3.8). Afterward, we can give the admission fee f using Eq. (5.5). For different arrival rates λ_2 of PU packets, we summarize the socially maximal profit U_s^* and the admission fee f in Table 4.

Table 4: Numerical results for admission fee f .

Arrival rate of PU packets λ_2	Socially maximum profit U_s^*	Admission fee f
0.01	0.4445	2.0748
0.02	0.2125	1.3196
0.03	0.0524	0.5071

6 Conclusions

Taking into account the transmission quality for both PU packets and SU packets, we proposed a novel channel aggregation strategy in cognitive radio networks. By constructing a two-dimensional Markov chain, we evaluated the system performance for the channel aggregation strategy, and validated the model analysis with numerical results. Taking a rational economic perspective, we established a system cost function to balance different performance

measures and optimize the channel aggregation intensity. Moreover, by discussing the Nash equilibrium and socially optimal behaviors for SU packets, we presented a pricing policy for SU packets, so that the system is socially optimized.

As a future work, we would consider the effect of imperfect channel sensing and also the procedure for accumulating transmission information relating to the interrupted packets using a new analysis model.

References

- [1] I. Akyildiz, W. Lee, M. Vuran and S. Mohanty, NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey, *Computer Networks* 50 (2006) 2127–2159.
- [2] D. Chen, Q. Zhang and W. Jia, Aggregation aware spectrum assignment in cognitive ad-hoc networks, in *Proc. of 3rd International Conference on Cognitive Radio Oriented Wireless Networks and Communications*, 2008, CD-ROM, 6 pages.
- [3] C. Do, N. Tran, M. Nguyen, C. Hong and S. Lee, Social optimization strategy in unobserved queueing systems in cognitive radio networks, *IEEE Communications Letters* 16 (2012) 1944–1947.
- [4] S. Jin, X. Yao and Z. Ma, A novel spectrum allocation strategy with channel bonding and channel reservation, *KSH Transactions on Internet & Information Systems* 9 (2015) 4034–4053.
- [5] J. Lei, V. Plat and F.Y. Li, Analysis on channel bonding/aggregation for multi-channel cognitive radio networks, in *Proc. of European Wireless Conference*, 2010, pp. 468–474.
- [6] C. Li, W. Chen, X. Zhang and D. Yang, Analysis and simulation for spectrum aggregation in LTE-advanced system, in *Proc. of 70th IEEE Vehicular Technology Conference Fall*, 2009, pp. 1–6.
- [7] F. Liu, Y. Xu, X. Guo, W. Zhang, D. Zhang and C. Li, A spectrum handoff strategy based on channel reservation for cognitive radio network, in *Proc. of 3rd International Conference on Intelligent System Design and Engineering Applications*, 2013, pp. 179–182.
- [8] Y. Lu, H. He, J. Wang and S. Li, Opportunistic spectrum access with channel bonding, in *Proc. of 4th International Conference on Communications and Networking in China*, 2009, CD-ROM, 5 pages.
- [9] Q. Peng, Y. Dong, W. Wu, H. Rao and G. Liu, Dynamic spectrum access scheme of variable service rate and optimal buffer-based in cognitive radio, *Communications and Network* 5 (2013) 232–237.
- [10] C. Pham, N.H. Tran, C.T. Do and C.S. Hong, Spectrum handoff model based on hidden Markov model in cognitive radio networks, in *Proc. of 28th International Conference on Information Networking*, 2014, pp. 406–411.
- [11] C. Tamal and M.L. Saha, An analytical framework for channel reservation scheme in cognitive radio network, in *Proc. of International Conference on Advances in Computing, Communications and Informatics*, 2013, pp. 127–132.

- [12] C. Wang and L. Wang, Analysis of reactive spectrum handoff in cognitive radio networks, *IEEE Journal on Selected Areas in Communications* 30 (2012) 2016–2028.
- [13] J. Wang, A. Huang, W. Wang and T. Quek, Admission control in cognitive radio networks with finite queue and user impatience, *IEEE Wireless Communications Letters* 2 (2013) 175–178.
- [14] Y. Zhao, S. Jin and W. Yue, Performance optimization of a dynamic channel bonding strategy in cognitive radio networks, *Pacific Journal of Optimization* 9 (2013) 679–696.

Manuscript received 18 July 2015
revised 15 March 2016
accepted for publication 15 July 2016

SHUNFU JIN
School of Information Science and Engineering
Yanshan University, Qinhuangdao, China
Key Laboratory for Computer Virtual Technology and System
Integration of Hebei Province
Yanshan University, Qinhuangdao, China
E-mail address: jsf@ysu.edu.cn

WUYI YUE
Department of Intelligence and Informatics
Konan University, Kobe, Japan
Email address: yue@konan-u.ac.jp

GANG LI
Qinhuangdao Port Co., LTD, Hebei Port Group Co. LTD
Qinhuangdao, China
Email address: l_g1989@sina.com