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OPTIMALITY CONDITION AND OPTIMAL CONTROL FOR A TWO-STAGE NONLINEAR DYNAMICAL SYSTEM OF MICROBIAL BATCH CULTURE*

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Abstract: In this paper, we consider a two-stage dynamical system for describing batch culture in microbial fermentation. An optimal control problem *involving* the two-stage dynamical system is proposed. The yield intensity of 1, 3-propanediol (1,3-PD) is taken as *the* performance index, the initial concentration of biomass, glycerol and the terminal time are as the control *variables*. Then the optimality condition is obtained based on the properties of the two-stage dynamical system. Inspired by the conception of *the Nelder-Mead* simplex, we presented a modified Nelder-Mead simplex search method to solve the optimal control problem and the optimality condition is used as a convergence criterion of the algorithm. The numerical results *show* the conclusion of this paper.

Key words: *Batch culture, optimal control, optimality condition, two-stage system, N-M simplex search method*

Mathematics Subject Classification: *49J15, 49M37, 65K10*

1 Introduction

1,3-Propanediol(1,3-PD) plays an important part in chemical industry. For its unique symmetrical structure, 1,3-PD can be used for synthesizing many cosmetics, lubricants, medicines and polymers such as polyesters and polyurethanes [11]. As it can be produced safely and cheaply, its microbial conversion has recently been *received* more and more attention. Since the 1980s, *there are many studies on glycerol bioconversion to 1,3-PD*, and glycerol bioconversion to 1,3-PD by *Klebsiella pneumoniae (K. pneumoniae)* which has been widely investigated due to its high productivity [1, 13, 22, 28–30]. The fermentation of glycerol by *K. pneumoniae* under anaerobic conditions is a complex bioprocess, *since* microbial growth is subjected to multiple inhibitions of substrate and products, such as glycerol, 1,3-PD, ethanol and so on [30].

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Several culture techniques have been practiced with regard to fermentation. Compared with continuous and *fed-batch* cultures, under some kind of mild conditions, glycerol dismutation in batch culture can obtain the highest production concentration and molar yield 1,3-PD to glycerol [6, 8, 19, 20] and the references cited therein. Relevant literature regarding batch culture includes references [24, 25], where pathway identification for a nonlinear system in batch culture is researched; reference [2], where robust suboptimal control of a microbial batch culture process is studied; references [27], where robustness analysis of nonlinear dynamical system in batch culture is investigated; reference [31], where strong stability of a nonlinear multi-stage dynamic system in batch culture of glycerol bioconversion to 1,3-propanediol is discussed; reference [9], where parameter identification for a nonlinear time-delay system in microbial batch fermentation is investigated; reference [26], where robust parameter identification using parallel global optimization for a batch nonlinear enzyme-catalytic time-delayed process presenting metabolic discontinuities is considered; reference [10], where bi-objective dynamic optimization of a nonlinear time-delay system in microbial batch process is researched; reference [18], where practical algorithm for stochastic optimal control problem about microbial fermentation in batch culture is studied; reference [23], where optimal control of a batch fermentation process with nonlinear time-delay and free terminal time and cost sensitivity constraint is taken into account.

In this paper, on the basis of the previous model and parameter in [5], we consider a two-stage dynamical system in batch culture. Then we take the yield intensity of 1,3-PD as the performance index, the initial concentration of biomass, glycerol and terminal time as the control vector. Based on the properties of the two-stage dynamical system and the solution of it, we obtain the optimality condition of the optimal control problem. Then the optimal condition is used as a convergence criteria of the optimization algorithm. Finally, we presented a modified Nelder-Mead simplex search algorithm to solve the optimal control problem.

This paper is organized as follows. In section 2, we simply review the two-stage nonlinear system of batch fermentation. In section 3, some properties of the solution to the system are proved. Then, an optimal control model and optimality conditions are proposed in section 4. Finally, we propose an algorithm and give the results in section 5, 6 and 7.

2 Two-stage Dynamical System in Batch Culture

In batch culture, a quantity of biomass and glycerol are added to the reactor only once and stirred uniformly under given conditions. During the process of the culture, the concentration of the glycerol decreases gradually and tends to zero finally. *Let I_n denotes the set of $\{1, \ldots, n\}$. Let $I := [0, t_f]$ denote the time interval and $t_f \in (0, +\infty)$ be the terminal moment of the batch culture.* According to the actual fermentation process, we make the following assumptions.

(A1) : During the process of batch fermentation, no medium is pumped inside or outside the reactor.
(A2) : The concentrations of reactants are uniform in the reactor.
Under the above assumptions, the simplified mass balances of biomass, substrate and prod-

ucts in batch culture [21] are written as follows

$$\begin{cases} \dot{x}_1(t) = \mu(t)x_1, & \\ \dot{x}_2(t) = -q_2(t)x_1, & t \in I, \\ \dot{x}_i(t) = q_i(t)x_1, \ i \in \{3, 4, 5\}, & \\ x_i(0) = x_{0i}, \ i \in I_5. & \end{cases} \tag{2.1}$$

where $x_1(t), x_2(t), x_3(t), x_4(t), x_5(t)$ denote the concentrations of biomass, glycerol, 1-3PD, acetic acid and ethanol concentrations at time t in reactor, and $x_{0i}, \ i \in I_5$, are the initial concentrations of biomass, glycerol, 1,3-PD, acetate and ethanol, respectively. Further the specific growth rate of cells μ, specific consumption rate of substrate q_2 and specific formation rates of products $q_i, i \in \{3, 4, 5\}$ are expressed by the following equations respectively,

$$\mu(t) = \mu_m \frac{x_2(t)}{x_2(t) + k_2} \prod_{i=2}^{5} \Big(1 - \frac{x_i(t)}{x_i^*}\Big)^{n_i}, \tag{2.2}$$

$$q_2(t) = m_2 + \frac{\mu(t)}{Y_2}, \tag{2.3}$$

$$q_i(t) = m_i + \mu(t)Y_i, \qquad i \in \{3, 4, 5\}, \tag{2.4}$$

where k_2 is the Monod saturation constant for substrate (in mmol L^{-1}); $x_i^*, i \in \{2, 3, 4, 5\}$, are, respectively, the critical concentrations of glycerol, 1,3-PD, acetate, and ethanol required for cell growth; $m_i, i \in \{2, 3, 4, 5\}$, are the maintenance terms of substrate consumption and product formation(in mmol $g^{-1}h^{-1}$) under substrate-limited conditions respectively; $Y_i, i \in \{3, 4, 5\}$, are the maximum product yields (in mmol g^{-1}). The maximum specific growth rate μ_m is $0.67(h^{-1})$. We collect these system parameters into a vector σ:

$$\sigma := [k_2, m_2, m_3, m_4, m_5, Y_2, Y_3, Y_4, Y_5] \in \mathbb{R}^9.$$

The following estimates obtained in [17] are used in this paper.

$$\sigma = [50, -2.2, -2.69, -0.97, 5.26, 0.0082, 67.69, 33.07, 11.66] \in \mathbb{R}^9.$$

Numerical results in [4] show that the system (2.1) can only describe the developmental and vegetation phases well. However, the errors between the experimental data and computational values are very large in the stationary phase. To formulate the process of batch fermentation better, we revise the model of the stationary phase according to the experimental data in the fermentation process. Let $t_g \in [0, t_f]$ be the moment after which the system reaches the stationary phase. Then the time inteval of culture process $[0, t_f]$ is divided into two phases, i.e., $[0, t_g)$ is the time interval of developmental and growth periods and $[t_g, t_f]$ is the one of stationary phase. So the two-stage nonlinear dynamical system can be formulated as follows,

$$\begin{cases} \dot{x_1}(t) = \mu(t)x_1, & \\ \dot{x_2}(t) = -q_2(t)x_1, & t \in [0, t_g), \\ \dot{x_i}(t) = q_i(t)x_1, & \\ x_j(0) = x_{0j}, \ j = 1, \dots, 5, & \end{cases} \tag{2.5}$$

$$\begin{cases} \dot{x_1}(t) = \mu(t)e^{-a_1(t-t_g)}x_1, & \\ \dot{x_2}(t) = -q_2(t)e^{-a_2(t-t_g)}x_1, & t \in [t_g, t_f], \\ \dot{x_i}(t) = q_i(t)e^{-a_i(t-t_g)}x_1,\ i = 3, 4, 5, & \\ x_j(t_g^+) = x_j(t_g),\ j = 1, \ldots, 5, & \end{cases} \tag{2.6}$$

where t_g is given by the experimental results and $x_j(t_g^+)$ denotes the right limit of concentration at time t_g.

Let $x(t) := (x_1(t), x_2(t), x_3(t), x_4(t), x_5(t))^\top$ be the concentrations of biomass, glycerol, 1,3- PD, acetate and ethanol at time t respectively. Then the upper and lower bounds of x are

$$x^* := (x_1^*, x_2^*, x_3^*, x_4^*, x_5^*)^\top = (10, 2039, 939.5, 1026, 360.9)^\top,$$

and

$$x_* := (x_{*1}, x_{*2}, x_{*3}, x_{*4}, x_{*5})^\top = (0.01, 200, 0.01, 0.01, 0.01)^\top,$$

respectively. Then the admission set of state variables is defined by

$$W_{ad} := \prod_{i=1}^{5} [x_{*i}, x_i^*].$$

The initial concentrations of 1,3-PD, acetate, and ethanol in the dynamic model (2.5) are given. The initial concentrations of biomass and glycerol together with the terminal time t_f are control variables to be optimized. And U_{ad} is the control *variables* admission set,

$$U_{ad} := \{(x_{01}, x_{02}, t_f)^\top | x_{01} \in [0.01, 1], x_{02} \in [200, 1700], t_f \in [2, 10]\}.$$

For convenience, we denote that

$$f^1(t, x(t, u)) := (\mu(t)x_1, -q_2(t)x_1, q_3(t)x_1, q_4(t)x_1, q_5(t)x_1)^\top, \tag{2.7}$$

and

$$\begin{aligned} f^2(t, x(t, u)) :=& \Big(\mu(t)e^{-a_1(t-t_g)}x_1, -q_2(t)e^{-a_2(t-t_g)}x_1, q_3(t)e^{-a_3(t-t_g)}x_1, \\ & q_4(t)e^{-a_4(t-t_g)}x_1, q_5(t)e^{-a_5(t-t_g)}x_1\Big)^\top. \end{aligned} \tag{2.8}$$

Then, the two-stage system (3) of the batch culture can be rewritten as

$$\begin{cases} \dot{x}(t) = f(t, x(t, u)), \\ x(0) = x_0, \end{cases} \tag{2.9}$$

where

$$f(t, x(t, u)) = \begin{cases} f^1(t, x(t, u)), & t \in [0, t_g], \\ f^2(t, x(t, u)), & t \in (t_g, t_f], \end{cases}$$

and $x_0 := (x_{01}, x_{02}, x_{03}, x_{04}, x_{05})^\top$ denotes the initial concentration.

From the equation (2.2) to (2.9), we can directly get that *for all* $u \in U_{ad}$ the function f in (2.9) is twice continuously differentiable and satisfies the linear growth condition, i.e., there *exists a* $K > 0$, such that

$$\|f(t, x(t, u))\| \leq\ K(\|x\| + 1). \tag{2.10}$$

3 Properties of the Two-stage System

To solve our problem, it is necessary to guarantee the existence, uniqueness and Lipschitz continuity of the solution to (2.9). In this section, we prove some properties of (2.9) and its solution. In view of the mechanism of bio-dissimilation of glycerol to 1,3-PD, the following assumptions hold.

(A3) : The concentrations of biomass, glycerol and three products are nonnegative during the process of the batch culture, that is, $x_i \geq 0, \ \forall \ i \in I_5$.
(A4) : The concentrations of biomass, glycerol and products can not exceed their critical values, namely, $x_i \leq x_i^*, \ i \in I_5$.

Proposition 3.1. *Under the assumptions (A3) and (A4), there exists a constant L, for all given $t \in I, \ x, y \in W_{ad}$ and $u, v \in U_{ad}$, so that function $f^i, \ i \in I_2$, defined by (2.7) and (2.8) satisfy the following condition*

$$\|f^i(t, x(t,u)) - f^i(t, y(t,v))\| \leq L(\|x-y\| + \|u-v\|), \tag{3.1}$$

where $\|\cdot\|$ denotes the Euclidean norm.

Proof. For all $t \in I, \ x, y \in S_0$ and $u, v \in U_{ad}$, we denote $y = x + \triangle x, v = u + \triangle u$. Then, by the differential mean value inequality, we obtain that

$$\begin{aligned} \|f^i(t, y(t,v)) - f^i(t, x(t,u))\| \leq & \Big\| \frac{\partial f^i}{\partial x}\Big(t, (x + \theta_1 \triangle x)(t,u)\Big)\Big\| \| \triangle x\| \\ & + \Big\| \frac{\partial f^i}{\partial u}\Big(t, (x + \triangle x)(t, u + \theta_2 \triangle u)\Big)\Big\| \| \triangle u\|. \end{aligned}$$

where $0 < \theta_1, \theta_2 < 1$. Let

$$A_{i1} := \frac{\partial f^i}{\partial x}\Big(t, (x + \theta_1 \triangle x)(t,u)\Big), \ A_{i2} := \frac{\partial f^i}{\partial u}\Big(t, (x + \triangle x)(t, u + \theta_2 \triangle u)\Big).$$

In view of (2.7) and (2.8), it is easy to show that $\|A_{ij}\|, i \in I_2, \ j \in I_2$ are bounded. Consequently, there exists $L_{ij} > 0$ such that $\|A_{ij}\| \leq \|L_{ij}\|$. Take $L = \max_{i,j}\{L_{ij}\}$, then (3.1) holds.

□

Proposition 3.2. *Given $x(t,u)$ and $y(t,v)$ are the solutions of system (2.9) for $u, v \in U_{ad}$ respectively, then $\exists \ \mathrm{K}_1, \ \mathrm{K}_2 > 0$, satisfy:*

$$\Big\| \frac{\partial f}{\partial x}(t, x(t,u)) - \frac{\partial f}{\partial x}(t, y(t,v))\Big\| \leq \mathrm{K}_1(\|x-y\| + \|u-v\|), \tag{3.2}$$

$$\Big\| \frac{\partial f}{\partial u}(t, x(t,u)) - \frac{\partial f}{\partial u}(t, y(t,v))\Big\| \leq \mathrm{K}_2(\|x-y\| + \|u-v\|), \tag{3.3}$$

Proposition 3.3. *Under the assumptions (A3) and (A4), for all $u \in U_{ad}$ and $t \in [0, t_f]$, the system (2.9) has a unique solution $x(t,u)$ which satisfies the following integral equation:*

$$x(t;u) = \begin{cases} \int_0^t f^1(s, x(s,u))ds + x_0, & t \in [0, t_g], \\ \int_{t_g}^t f^2(s, x(s,u))ds + x(t_g, u), & t \in (t_g, t_f]. \end{cases} \tag{3.4}$$

Furthermore, $x(t,u)$ is Lipschitz continuous in u on U_{ad}.

Proof. Since $f^i(t, x(t, u)),\ i \in I_2$ is Lipschitz continuous with respect to x in W_{ad}, for a given u, the system (2.9) has a unique solution by the theory of differential equations. Obviously, the solution $x(t, u)$ is continuous in I. Now, we prove that $x(t; u)$ is Lipschitz continuous in u on U_{ad}.

Case 1. The case for $t \in [0, t_g]$.

On the basis of Property (3.1), Property (3.2) and function (3.4), we obtain that, for all $u, v \in U_{ad}$,

$$\|x(t,u) - x(t,v)\| \leq Lt_g\|u - v\| + L\int_0^t \|x(s,u) - x(s,v)\|ds.$$

By Bellman Gronwall inequality, the following

$$\|x(t,u) - x(t,v)\| \leq Lt_g\|u - v\|e^{(Lt_g)},$$

holds.

Case 2. The case for $t \in (t_g, t_f]$.

Similar to Case 1, the following

$$\|x(t,u) - x(t,v)\| \leq L(t_g e^{Lt_g} + (t_f - t_g))e^{L(t_f - t_g)}\|u - v\|,$$

holds. According to the above cases, we complete the proof.

□

Proposition 3.4. *Under the assumptions (A3) and (A4), the solution (3.4) is bounded, that is , there exists an $M > 0$ such that, for any $t \in I$,*

$$\|x(t,u)\| \leq M,\ \forall\ u \in U_{ad}. \tag{3.5}$$

Proof. According to (2.10) and Bellman Gronwall inequality, we can obtain our desired result.

□

For given $x_0 \in W_{ad}$, we define the set of solutions to the system (2.9) as follows,

$$S_1 := \{x(t,u) \in R^5 | x(t,u) \text{ is a solution to the system (2.9) } \forall\ u \in U_{ad}\}. \tag{3.6}$$

In view of the compactness of $U_{ad} \subseteq R^3$ and the continuity of the mapping from $u \in U_{ad}$ to $x(t, u)$, we have the following result.

Proposition 3.5. *The set S_1 defined by (3.6) is compact in $C^1([0, t_f]; R^5)$.*

4 Optimal Control Problem and Optimality Condition

Let $x_i(t,u),\ i \in I_5$, be the solution of (2.9) corresponding to the control *vector* $u := (x_{01}, x_{02}, t_f)$. Then our goal is to maximize the yield of 1,3-PD in microbial fermentation. Thus, we need to choose the control vector u to maximize the following objective function $J(u)$:

$$\begin{aligned} \text{(P)} \qquad & J(u) = \frac{x_3(t_f,u)}{t_f} \\ \text{s,t} \quad & t_f \in I, \\ & u \in U_{ad}. \end{aligned} \tag{4.1}$$

According to [3], we make some notes of the state constraint set W_{ad} as follows

$$\begin{aligned} \varphi_j(t,u) &:= x_j(t,u) - x_j^*,\ j \in I_5, \\ \varphi_{j+5}(t,u) &:= -x_j(t,u) + x_{*j},\ j \in I_{10}, \\ f_j(u) &:= \max_{t\in I}\{\varphi_j(t,u)\},\ j \in I_{10}, \\ f^0(u) &:= -J(u). \end{aligned} \tag{4.2}$$

So the optimal control problem can be equivalent to *the following problem*:

$$\textbf{(OCP)} \qquad \min\{f^0(u) | u \in U_{ad},\ f_j(u) \le 0, j \in I_{10}\}. \tag{4.3}$$

Based on the theorem 4.1.5 in [15] and (4.2), we know that the directional derivatives $df_j(u, \delta u)$ $j \in I_{10}$, *exist and are* given by

$$\begin{aligned} df_j(u,\delta u) &= \max_{t\in T_j(u)} \langle \nabla_u \varphi_j(t,u), \delta u\rangle, \\ &= \max_{z\in \partial f_j(u)} \langle z, \delta u\rangle, \end{aligned}$$

where

$$T_j(u) := \{t \in I | \varphi_j(t,u) = f_j(u)\},$$

and

$$\partial f_j(u) := co_{t\in T_j(u)}\{\nabla_u \varphi_j(u,t)\}.$$

Let $u^* := (x_{01}^*, x_{02}^*, t_f^*)$ be the optimal solution of **(OCP)**, and $\psi(u) := \max\{f_j(u),\ j \in I_{10}\}$ then from [3], we have

$$d\psi(u^*, u-u^*) = \max_{j\in \overline{q}(u^*)} df_j(u^*, u-u^*).$$

Further, for convenience when getting the optimal conditions, we define $F(u)$ as

$$F(u) := \max\{f^0(u) - f^0(u^*), \psi(u)\},$$

then it follows from the properties and the solution of the system that:

Theorem 4.1. *If $u^* \in U_{ad}$ is the optimal solution of problem OCP, then $F(u)$ researches the minimum at u^*.*

Theorem 4.2. *If $u^* \in U_{ad}$ is the optimal solution of problem OCP, then*

$$dF(u^*, u-u^*) \ge 0, \quad \forall u \in U_{ad}. \tag{4.4}$$

5 Algorithm

Based on the Nelder-Mead *(N-M)* simplex search, we developed a modified N-M simplex method to solve Problem **OCP**. The N-M simplex method, in some cases, is more convenient to solve optimal control problems than another gradient-based optimization methods.

The N-M simplex method uses the concept of a simplex, which is a special polytope of $N+1$ vertices in N dimensions [14]. That is to say, Nelder-Mead maintains a set of $N+1$ test points arranged as a simplex in N dimensions. It then extrapolates the behavior of the objective function measured at each test point, in order to find a new test point and to replace one of the old test points with the new one, and so the technique progresses. The simplest approach is to replace the worst point with a point reflected through the centroid of the remaining N points. If this point is better than the best current point, then we can try stretching exponentially out along this line. *O*n the other hand, if this new point isn't much better than the previous value, then we are stepping across a valley, so we shrink the simplex towards a better point.

Firstly, we calculate the *function* values at the initial vertices. Considering the minimization case, we replace the vertex with the highest function value by a better one at each time. The replacement process consists of four operations: reflection, expansion, contraction and reduction. After each replacement, the simplex *becomes* better and be closer to the locally optimal point [7, 12]. As the number of initial vertices are changeable according to the problems, this method is extremely flexible when solving the **OCP**.

In this paper, Problem **OCP** is an optimization problem subject to the two-stage dynamical system (2.9). We use the 4th Runge-Kutta method to solve the system (2.9) and the modified simplex method is as follows.

Step 1. Generate the initial simplex with K vertices randomly and the control vertices (x_{01}, x_{02}, t_f) denoted by $P^1, \ldots, P^K$, where $P^i = (p_1^i, p_2^i, p_3^i) \in U_{ad}, i \in I_K$. Solve the system (2.9) by the forth order Runge-Kutta algorithm, then to compute the function value J^i of the *ith* vertex.
Step 2. Find the best and the worst vertex P^b, P^w, and the function values J^b, J^w, respectively, by ordering the values J^i. *If* $\dfrac{|J^w - J^b|}{|J^b|} < \bar{\epsilon}$ *(where* $\bar{\epsilon}$ *is the convergence tolerance of the relative deviation corresponding to the cost function) or the result in Theorem 4.2 holds, then output the corresponding result and we have a successful exit; otherwise, go to Step 3.*
Step 3. Calculate P^c, the centroid of all points except P^w, by the following formula

$$P^o = (P^1 + \ldots + P^{w-1} + P^{w+1} + \ldots + P^K)/(K-1). \tag{5.1}$$

Then we get the reflected point P^r defined by $P^r = P^o + \alpha(P^o - P^w),\ \alpha > 0$.
Step 4. If $P^r \notin U_{ad}$, then we have

$$p_j^r = \begin{cases} u_j^*, & \text{if } p_j^r \geq u_j^*, \\ u_{j*}, & \text{if } \ p_j^r \leq u_{j*}, \end{cases} \qquad j \in I_3. \tag{5.2}$$

where (u_1^*, u_2^*, u_3^*) and (u_{1*}, u_{2*}, u_{3*}) *denote* the upper and lower bound of (x_{01}, x_{02}, t_f), respectively.
Step 5. If the reflected *point* is the best so far, i.e., $J^r > J^b$, then compute the expansion point $P^e = P^o + \beta(P^o - P^w),\ \beta > \alpha$, else go to *S*tep 7.
Step 6. If the expanded point is better than the reflected point, $J^e > J^r$, then we can obtain a new simplex by replacing the worst point P^w with the expanded point P^e, and go

to Step 2*; otherwise,* we obtain a new simplex by replacing the worst point P^w with the reflected point P^r, and go to Step 2.

Step 7. *Compute* the contracted point $P^c = P^o - \delta(P^o - P^w),\ \delta \in (0,1)$*. If* $J^c > J^w$, go to Step 2*; otherwise,* go to Step 8.

Step 8. For all but the best point, replace the point with $P^i = P^b + \sigma(P^i - P^b)$, go to Step 2.

Then we get the optimal parameter vector.

6 Numerical Results

On the basis of the model and algorithm mentioned above, we have programmed the software and applied it to the optimal control problem of microbial fermentation in batch culture. The system parameters are listed in Table 1 [5].

Table 1: *The system parameters* in *(2.2)-(2.6)*

m_2	m_3	m_4	m_5	Y_2	Y_3	Y_4	Y_5	K
0.0100	-3.9472	2.1098	-0.1830	0.0165	41.2584	4.5410	3.0460	39.68
n_2	n_3	n_4	n_5	a_1	a_2	a_3	a_4	a_5
0	1	5	1	1.804	0.23	0.551	0.12	0

And the parameters in the algorithm are as follows: $K = 5, \alpha = 1, \beta = 1.6, \gamma = 0.5, \delta = 0.5, \epsilon = 0.001, \bar{\epsilon} = 0.01$. Then, by the N-M simplex search method, the optimal control vector and the performance index are $(0.98793, 785.828, 4.94116)^T$ and 56.266, respectively. The change of the concentration of 1,3-PD with respect to t is plotted in Figure 1.

From the image above we can see that the concentration of 1,3-PD keeps increasing at first and reaches the highest rate of growth at about $6.7h$. While at the end of the fermentation, it begins to decrease slowly.

7 Conclusions and Future Works

In this paper, we first introduce a two-stage dynamical system in batch culture and discuss the properties of its solution and optimality condition. Then we propose a modified N-M simplex search method to solve the optimal control problem. The final results showed how the concentration of 1,3-PD varied with respect to time and it wasn't increasing all the time but reached the maximum at about $6.7h$.

Next, it is intended to study models of continuous culture in which the fermentation covers both extracellular and intracellular environments and compare with the results in batch culture.

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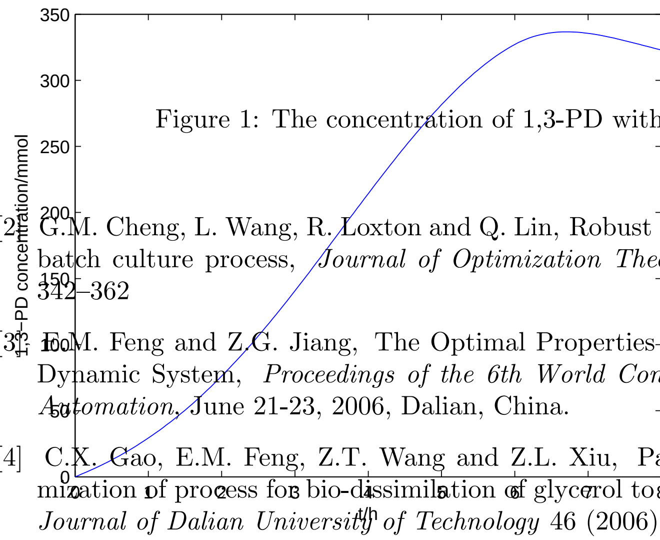

Figure 1: The concentration of 1,3-PD with respect to time t

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