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LEVENBERG-MARQUARDT METHOD FOR SMART GRID WITH CONTROLLABLE SUPPLY

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Abstract: Currently, in the context of carbon reduction and harmonious advance in the environment, environment-friendly smart grid is developing rapidly. Among the existing smart grid models, there exists little research about the smart grid with controllable supply, so this paper aims to study the problem of smart grid with controllable supply. The model of smart grid with controllable supply is proposed, and the real-time price of smart grid with controllable supply can be gotten by the Karush-Kuhn-Tuker(KKT) conditions of interval optimization. Levenberg-Marquardt method is applied to solve the problem of smart grid with controllable supply, and also maximize social welfare. Finally, the simulation results and final remarks are also given.

Key words: *smart grid with controllable supply; real-time price; interval optimization; Levenberg-Marquardt method*

Mathematics Subject Classification: *90C90*

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1 Introduction

Smart grid is the inevitable result of economic and technological development, which specifically employs advanced technique to increase the performance for electricity power system in power utilization, power supply quality and reliability. The foundation of smart grid is divided into data transfering, calculation and control technique between several electricity providing units. Smart grid has excellent features of energy-saving, reliable, self-healing, fully controllable and asset efficient. So in recent years the theory and method of smart grid with the characteristics of high-quality, reliability, self-healing, interaction, security and environmental protection have been studied (one can see [6, 10, 17, 19, 21, 26]), such as, a novel real-time price model was developed in the case of DSM programs in [19], which encourage users to desire energy consumption behaviors. [10] constructed a social welfare maximization model based on the complementarity of energy between micro-grid and large-scale grid. In [6, 17, 26], the optimal Lagrange multiplier for the KKT optimal conditions was considered as the real-time price. In [26], the authors smoothed the KKT conditions by using the smooth NCP function. A new smooth conjugate gradient method to solve real-time price for smart grid based on maximizing social welfare was employed. And the final results showed the rationality, feasibility and effectiveness of the model. In [21], the authors selected a kind of shadow price, whose weighted norm was minimum when Lagrange multipliers were not unique. And the penalty function method and accelerated gradient method were considered.

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On the other hand, interval optimization has been widely studied, especially its optimality conditions. More theoretical research work of interval optimization can be found in [2,7,22,23,25,27] and the references therein. Interval optimization is increasingly used in practice such as economic planning, energy development, engineering design, environmental protection and other fields (one can see [9,18,29]). So, it is natural to consider whether the real-time price for smart grid based on maximizing social welfare can be solved by interval optimization? Can KKT conditions on interval optimization be applied to solve smart grid with controllable supply? This is also the motivation of this paper.

The remaining paper is organized as follows. In Section 2, relevant preliminaries about interval optimization and smart grid problem are introduced. In Section 3, we give the model of smart grid with controllable supply based on maximizing social welfare. The KKT optimal conditions for smart grid with controllable supply based on interval optimization are used to get real-time price. By transforming KKT conditions into an equivalent nonsmooth optimization problem, we rewrite it into a novel unconstrained optimization problem. In Section 4, we apply Levenberg-Marquardt method to get the optimal real-time electricity price and the optimal consumption of the customer-side. And simulation results are also given. Finally, we make final remarks in Section 5.

2 Preliminaries

Followings are some relevant preliminaries about interval optimization and smart grid model which can be found in [6,16,17,19,21–23,26].

Definition 2.1. $\Lambda = [\omega^L, \omega^U]$ and $\Xi = [\psi^L, \psi^U]$ are closed intervals in R. $\Lambda \preceq_{LU} \Xi \Leftrightarrow \omega^L \leq \psi^L$ together with $\omega^U \leq \psi^U$. On the other hand, $\Lambda \prec_{LU} \Xi \Leftrightarrow \Lambda \preceq_{LU} \Xi$ with $\Lambda \neq \Xi$.

$\Lambda \prec_{LU} \Xi$ if and only if one of the following conditions holds

$(i) \begin{cases} \omega^L < \psi^L \\ \omega^U \leq \psi^U \end{cases}$ (ii) $\begin{cases} \omega^L \leq \psi^L \\ \omega^U < \psi^U \end{cases}$ (iii) $\begin{cases} \omega^L < \psi^L \\ \omega^U < \psi^U \end{cases}$

Given $\Gamma^L(x) : R^n \to R$, $\Gamma^U(x) : R^n \to R$, and $\Gamma^L(x) < \Gamma^U(x)$.

Definition 2.2. The interval-valued function $\Gamma(x) = [\Gamma^L(x), \Gamma^U(x)]$ is differentiable at $x^* \in R^n \Leftrightarrow \Gamma^L$ and Γ^U are differentiable at x^*.

Interval optimization problem in general is as follows [23]:

$$\begin{array}{ll} \min & \Gamma(x) \\ s.t. & x \in \Theta = \{x | h_j(x) \leq 0, x \geq 0, j = 1, 2, \ldots, m-1, m\}, \end{array} \tag{2.1}$$

where $\Gamma(x) = [\Gamma^L(x), \Gamma^U(x)] : R^n \to R$ and $h_j(x) : R^n \to R$ is real-valued function, $j = 1, 2, \ldots, m-1, m$.

Definition 2.3. x^* is a feasible solution for (2.1). If there exists no $\Gamma(\overline{x}) \preceq_{LU} \Gamma(x^*)$, where $\overline{x} \in \Theta$, x^* is called a nondominated solution for (2.1) i.e. in this situation, $\Gamma(x^*)$ is konwn as the nondominate objective function value for Γ.

Smart grid based on maximizing social welfare is given as

$$\begin{array}{ll} \max & (\sum_{i=1}^{N} U_i(x_i^1, \omega_i^1) - \Upsilon_1(\hbar_1)) + \ldots + (\sum_{i=1}^{N} U_i(x_i^K, \omega_i^K) - \Upsilon_K(\hbar_K)) \\ s.t. & \sum_{i=1}^{N} x_i^k \leq \hbar_k, \quad \hbar_k^{min} \leq \hbar_k \leq \hbar_k^{max}, \quad k = 1, \ldots K, \\ & x_i^k \geq 0. \end{array} \tag{2.2}$$

In (2.2), there are N electric power customers and one electric power supplier, and each power cycle is divided into K periods. x_i^k denotes the electricity consumption of user i in period k. $\hbar_k$ denotes the production efficiency of the electric power supplier in period k. $\Upsilon_k(\hbar_k)$ denotes the cost for producing power $\hbar_k$ in time division k for a power provider. $\hbar_k^{min}$ represents minimum electricity provided by an electric power provider in period k, and $\hbar_k^{max}$ represents maximum electricity provided by an electric power provider in period k. $U(x_i^k, \omega_i^k)$ is the utility function which is used to express the satisfaction of electricity user i in period k. In additon, $\omega_i^k \geq 0$, to represent the experience values of different users.

3 Smart Grid with Controllable Supply

Firstly, we give the model of smart grid with controllable supply, and then give the KKT conditions for it. We present the model for smart grid with controllable supply as follows

$$\begin{array}{ll} \min & \left[\Re^L(z), \Re^U(z)\right] \\ s.t. & \sum_{i=1}^{N} x_i^k - \hbar_k \leq 0, i = 1, \ldots, N, k = 1, \ldots K, \\ & x_i^k \geq 0, \end{array} \tag{3.1}$$

where

$$\begin{aligned} \Re^L(z) &= (\Upsilon_1(\hbar_1^{min}) - \sum_{i=1}^{N} U_i(x_i^1, \omega_i^1)) + \ldots + (\Upsilon_K(\hbar_K^{min}) - \sum_{i=1}^{N} U_i(x_i^K, \omega_i^K)), \quad (3.2) \\ \Re^U(z) &= (\Upsilon_1(\hbar_1^{max}) - \sum_{i=1}^{N} U_i(x_i^1, \omega_i^1)) + \ldots + (\Upsilon_K(\hbar_K^{max}) - \sum_{i=1}^{N} U_i(x_i^K, \omega_i^K)). \end{aligned}$$

In (3.1) and (3.2), $z = (x_1^1, \ldots x_N^1, x_1^2, \ldots x_N^2, \ldots, x_1^K, \ldots x_N^K, L_1, L_2, \ldots L_K)^T$, x_i^k denotes the electricity consumption of user i in period k. $\omega_i^k \geq 0$. $\hbar_k$ represents production efficiency for the power provider in time division k. $\hbar_k^{min}$ represents minimum electricity provided in time division k, and $\hbar_k^{max}$ represents maximum electricity provided in time division k.

Here, the utility function is

$$U(x, \omega) = \begin{cases} \ln(\omega x) + d & , x > 0 \\ 0 & , x = 0 \end{cases}$$

ω and d are two non-negative parameters according to different demands and satisfaction of electricity consumption. And the cost function is choosed as

$$\Upsilon_k(\hbar_k) = a_k \hbar_k^2 + b_k \hbar_k + c_k,$$

where $a_k > 0$ and $b_k > 0$, $c_k > 0$ are pre-determined parameters.

From [23], we know that the nondominated solution for minimization problem (3.1) is the optimal solution for minimization problem as follows:

$$\begin{array}{ll} \min & \Re(z) = \Re^L(z) + \Re^U(z) \\ s.t. & \tilde{h}_k(z) = \sum_{i=1}^{N} x_i^k - \hbar_k \leq 0, i = 1, \ldots, N, k = 1, \ldots K, \\ & x_i^k \geq 0, \end{array} \tag{3.3}$$

where $\Re^L(z)$, $\Re^U(z)$, x_i^k, $\tilde{h}_k(z)$, $\hbar_k$ are defined as (3.1) and (3.2).

According to [5], we know $\tilde{h}_k(z), k = 1, \ldots, K$ in (3.3) satisfying the linear independent constraint specification.

Now, denote $h_k(z) = -\tilde{h}_k(z)$, we have the theorem as follows.

Theorem 3.1. *Assume z^* is an optimal solution for problem* (3.3) *(also a nondominated solution for problem* (3.1)*), and $\Re$, $h_k, k = 1, \ldots, K$, have differentiability at z^*. And for h_k, $k = 1, \ldots, K$ satisfying the linear independent constraint qualification, then we can get multipliers $\mu_k \geq 0$ for $k = 1, \ldots, K$ satisfying*

$$\begin{cases} \nabla\Re^L(z^*) + \nabla\Re^U(z^*) - \mu_1\nabla h_1(z^*) - \mu_2\nabla h_2(z^*) - \ldots - \mu_K\nabla h_K(z^*) = 0, \\ \hfill \mu_k h_k(z^*) = 0, \\ \hfill \mu_k \geq 0, \\ \hfill h_k(z^*) \geq 0, \end{cases} \tag{3.4}$$

where $k = 1, \ldots, K-1, K$.

Proof. According to (3.2), $\Re^L(z)$ and $\Re^U(z)$ have differentiability at z^*, which can be said $\Re(z) = \Re^L(z) + \Re^U(z)$ has differentiability at z^*. Next, we give a proof by contradiction. Assume

$$\begin{array}{rcl} \nabla\Re(z^*)d & < & 0, \\ \nabla h_i(z^*)d & \leq & 0 \quad i \in \emptyset(z^*), \end{array} \tag{3.5}$$

where $d \in R^n$, $\emptyset(z^*) = \{i | h_i(z) = 0\}$ be indicator set with effective constraints. Since $h_k, k = 1, \ldots, K$ satisfy the linear independent constraint qualification and $\Re(z)$ has differentiability at z^*, we have

$$\begin{aligned} \Re(\natural(t)) &= \Re(z^*) + \nabla\Re(z^*)^T(\natural(t) - z^*) + \|\natural(t) - z^*\| \cdot \delta(\natural(t), z^*) \\ &= \Re(z^*) + \nabla\Re(z^*)^T(\natural(t) - \natural(0)) + \|\natural(t) - \natural(0)\| \cdot \delta(\natural(t), \natural(0)) \\ &= \Re(z^*) + \nabla\Re(z^*)^T(\frac{\natural(0+t) - \natural(0)}{t}) + \|\natural(t) - \natural(0)\| \cdot \delta(\natural(t), \natural(0)) \end{aligned}$$

where $\delta(\natural(t), \natural(0)) \to 0$ as $\|\natural(t) - \natural(0)\| \to 0$. So, as $t \to 0^+$, $\|\natural(t) - \natural(0)\| \to 0$ and $\frac{\natural(0+t)-\natural(0)}{t} \to \natural'_+(0) = \beta d$, where $\beta > 0$.

Since $\nabla\Re(z^*)^T d < 0$, we have $\Re(\natural(t_0)) < \Re(z^*)$ for an arbitrarily small $t_0 > 0$, which is contrary to z^* being optimal solution for problem (3.3). Therefore, assumption (3.5) does not hold and we can get that there exists no d satisfying the inequalities in (3.5). By Tucker's theorem in [24], we have

$$\lambda\nabla\Re(z^*) - \sum_{i\in\emptyset(z^*)} \overline{\mu}_i \nabla h_i(z^*) = 0$$

or equivalently

$$\nabla\Re(z^*) - \sum_{i\in\emptyset(z^*)} \mu_i \nabla h_i(z^*) = 0,$$

where $\lambda > 0$ and $\overline{\mu}_i \geq 0$ for $i \in \emptyset(z^*)$. Denote $\mu_i = \overline{\mu}_i/\lambda$, thus, the proof is completed. □

Define Φ maps from $R^n \times \Omega$ to R^n as

$$\Phi(z) = \begin{pmatrix} \phi(\mu_1, h_1(z)) \\ \phi(\mu_2, h_2(z)) \\ \vdots \\ \phi(\mu_K, h_K(z)) \end{pmatrix},$$

where ϕ is NCP function with properties as follows [3],

$$\phi(\varrho, \varpi) = 0 \Leftrightarrow \varrho \geq 0, \varpi \geq 0, \varrho\varpi = 0,$$

where $\varrho, \varpi \in R$.

From [3], we know that the complementary problem aims to find $x \in R^n$, satisfying

$$x \geq 0, \quad \Im(x) + \kappa \geq 0, \quad \eta^T(\Im(x) + \kappa) = 0,$$

where $\Im(x)$ maps from R^n to R^n and $\kappa \in R^n$.

Accordingly,

$$\mu \geq 0, \quad h(z) \geq 0, \quad \mu^T h(z) = 0$$

is a complementarity problem, where $\mu = (\mu_1, \mu_2, \ldots, \mu_K)^T$, $h(z) = (h_1(z), \ldots, h_K(z)).^T$

Thereafter, we can transform $\mu_k \geq 0$, $h_k(z) \geq 0$, $\mu_k h_k(z) = 0$. $k = 1, \ldots, K$ in (3.4) into

$$\Phi(\mu, h(z)) = \begin{pmatrix} \phi(\mu_1, h_1(z)) \\ \phi(\mu_2, h_2(z)) \\ \vdots \\ \phi(\mu_K, h_K(z)) \end{pmatrix} =0,$$

where

$$\phi(\mu_i, h_i(z)) = \frac{\mu_i + h_i(z)}{2} - \frac{|\mu_i - h_i(z)|}{2}, \quad i = 1, \ldots, K, \tag{3.6}$$

ϕ is NCP function [20].

Let $G : R^n \to R^n$ be a locally-Lipschitzian function. The B-subdifferential of G at x is

$$\partial_B G(x) = \{V \in R^{m\times n} | \exists x_k \subseteq D_G : \{x_k\} \to x, G'(x_k) \to V$$

where D_G is the differentiable points set and $G'(x)$ is the Jacobian of G at a point $x \in R^n$.

The Clarke generalized Jacobian of G is defined as

$$\partial G(x) = conv\{V \in R^{m\times n} | \exists x_k \subseteq D_G : \{x_k\} \to x, G'(x_k) \to V\},$$

we have

$$\partial_C G(x) = \partial G_1(x) \times \partial G_2(x) \times \ldots \times G_m(x)$$

denoting the C-subdifferential of G at x.

Definition 3.2. G is semi-smooth at x if

$$\lim_{V \in \partial G(x+th'), h' \to h, t \to 0^+} Vh'$$

exists for any $h \in R^m$.

Proposition 3.3. *$\phi(\mu, h(z))$ is semismooth, if $\phi(\mu, h(z))$ is defined as (3.6), and $\partial \Phi_C(\mu, h(z))$ is a K-dimensional diagonal matrix with diagonal elements of $\{\frac{1}{2} - \frac{1}{2}v, \frac{1}{2} - \frac{1}{2}v\}$, where*

$$v \in \partial|\mu_i - h_i(z)| = \begin{cases} 1 & , \mu_i > h_i(z), \quad i = 1, \ldots, K, \\ -1 & , \mu_i < h_i(z), \quad i = 1, \ldots, K, \\ [-1, 1] & , \mu_i = h_i(z), \quad i = 1, \ldots, K. \end{cases}$$

Proof. Since $\frac{\mu+h(z)}{2}$ is smooth function, and from [15], $\frac{|\mu-h(z)|}{2}$ is semismooth function, $\phi(\mu, h(z))$ is semismooth function. Further, owing to $diag(\partial|\mu_i - h_i(z)|)$ a diagonal matrice, $i = 1, \ldots, K$, we can get $\partial\phi(\mu, h(z))$. We complete the proof. $\square$

Therefore, solving (3.4) is transformed into solving the equation system as follows

$$\Pi(z) = 0, \tag{3.7}$$

where

$$\Pi(z) = \begin{pmatrix} \nabla \Re_k^L(z) + \nabla \Re_k^U(z) - \mu_1 \nabla h_1(z^*) - \mu_2 \nabla h_2(z^*) - \ldots - \mu_K \nabla h_K(z^*) \\ \Phi(\mu, h(z)) \end{pmatrix}.$$

So solving (3.7) is equivalent to solving the following optimization problem

$$\min B(z) = \frac{1}{2}\|\Pi(z)\|^2, \quad z \in R^{K(N+2)}. \tag{3.8}$$

The function $\Pi(z)$ constructed via $\Phi(z)$ is semismooth on $R^{K(N+2)}$. If $\Pi(z)$ is locally Lipschitz continuous for $z \in U(z^*)$, where $z^* \in R^{K(N+2)}$, then $\Pi(z)$ is semismooth at z^*. $B(z)$ in (3.8) has continuous differentiability on $R^{K(N+2)}$ and

$$\nabla B(z) = \begin{pmatrix} \nabla^2 \Re_k^L(z) + \nabla^2 \Re_k^U(z) - \sum_{k=1}^K \mu_k \nabla^2 h_k(z) \\ V_\phi \end{pmatrix}^T \Pi(z), \tag{3.9}$$

where $V_\phi \in \partial_C \Phi(\mu, h(z))$.

4 Levenberg-Marquardt Method

In this section, the transformation problem (3.8) is considered. As the problem involves the structure for nonsmooth equations, the Levenberg-Marquardt method, as an important optimization method, has important applications in solving nonsmooth optimization problems and related problems (such as [1, 4, 8, 11–14, 28]).

Denote d_t as the search direction and α_t as step size, $\tilde{\tau}_t$ denotes parameter.

Algorithm 1

Initial: Given starting point $z_0 \in R^{K(N+2)}$ and $\tilde{\tau}_0 > 0$. Choose paraments $\rho, \sigma, 0 < \rho, \sigma < 1$. Let $0 < \varepsilon << 1$, $\nabla B(z_t) = \nabla \Pi(z_t)^T \Pi(z_t)$. Set $t := 0$.

Step 0: If $\|\nabla B(z_t)\| < \varepsilon$, stop.

Step 1: Solve the following equation

$$((\nabla \Pi(z_t)^T \nabla \Pi(z_t) + \tilde{\tau}_t I) d_t = -\nabla B(z_t),$$

Set $\tilde{\tau}_0 := \|\Pi(z_0)\|^2$, where

$$\tilde{\tau}_{t+1} := \begin{cases} 0.1\tilde{\tau}_t & , \eta_t > 0.75, \\ \tilde{\tau}_t & , 0.25 \leq \eta_t \leq 0.75, \\ 10\tilde{\tau}_t & , \eta_t \leq 0.25. \end{cases}$$

and $\eta_t = \frac{\Pi(z_{t+1}) - \Pi(z_t)}{(\nabla \Pi(z_t)^T \Pi(z_t))^T d_t + \frac{1}{2} d_t^T (\nabla \Pi(z_t)^T \Pi(z_t)) d_t}$.

Step 2: Let m_t be the minimum nonnegative integer that satisfies the following inequality

$$B(z_t + \rho^{m_t} d_t) \leq B(z_t) + \sigma \rho^{m_t} \nabla B(z_t)^T d_t,$$

where $\alpha_t = \rho^{m_t}$.

Step 3: Set $z_{t+1} = z_t + \alpha_t d_t$. $t := t + 1$, return to Step 0.

Now, we get the convergent analysis of the algorithm.

Theorem 4.1. *Suppose that $\{z_t\}$ is generated by Algorithm 1. And $\{z_t, \tilde{\tau}_t\} \to \{z^*, \tilde{\tau}^*\}$, if $\{z^*, \tilde{\tau}^*\}$ satisfies $\nabla \Pi(z^*)^T \nabla \Pi(z^*) + \tilde{\tau}^* I$ positive definite, z^* is the stationary point for* (3.8).

Proof. Since $\tilde{\tau}_t > 0$ and d_t is descent direction, we know that $\{z_{t_j}\} \to z^*$ satifying

$$\nabla \Pi_{t_j}^T \nabla \Pi_{t_j} \to \nabla \Pi(z^*)^T \nabla \Pi(z^*),$$

where $\tilde{\tau}_{t_j} \to \tilde{\tau}^*$.

And $v(\nabla \Pi(z^*)^T \nabla \Pi(z^*) + \tilde{\tau}^* I) v^T > 0, v \in R^n, v \neq 0$, if $\nabla B(z^*) \neq 0$, we have

$$d_{t_j} \to d^* = -[\nabla \Pi(z^*)^T \nabla \Pi(z^*) + \tilde{\tau}^* I]^{-1} \nabla \Pi(z^*)^T \Pi(z^*),$$

since d^* is descent direction of z^*, there exists $m^* \geq 0$ such that

$$B(z^* + \rho^{m^*} d^*) < B(z^*) + \sigma \rho^{m^*} \nabla B(z^*)^T d^*,$$

where $\rho \in (0, 1)$.

When j is sufficiently large, and $z_{t_j} \to z^*$, we get

$$B(z_{t_j} + \rho^{m^*} d_{t_j}) < B(z_{t_j}) + \sigma \rho^{m^*} \nabla B(z_{t_j})^T d_{t_j}.$$

By line search we know $m^* \geq m_{t_j}$, i.e.,

$$\begin{aligned} B(z_{t_j+1}) &= B(z_{t_j} + \rho^{m_{t_j}} d_{t_j}) \\ &\leq B(z_{t_j}) + \sigma \rho^{m_{t_j}} \nabla B(z_{t_j})^T d_{t_j} \\ &\leq B(z_{t_j}) + \sigma \rho^{m^*} \nabla B(z_{t_j})^T d_{t_j}, \end{aligned} \tag{4.1}$$

so for j (sufficiently large), we have

$$B(z_{t_j+1}) \leq B(z_{t_j}) + \sigma\rho^{m^*}\nabla B(z_{t_j})^T d_{t_j}.$$

Also since when $j \to \infty, z_{t_j+1} \to z^*$, we get

$$\lim_{j\to\infty} B(z_{t_j+1}) = \lim_{j\to\infty} B(z_{t_j}) = B(z^*).$$

By (4.1), we get

$$B(z^*) \leq B(z^*) + \sigma\rho^{m^*}\nabla B(z^*)^T d^*,$$

which is contradictory with $\nabla B(z^*)^T d^* < 0$. So $\nabla B(z^*) = 0$. The proof is completed. □

Corollary 4.2. *Since Q in* (3.9) *is nonsingular. By Theorem 3.1 and Theorem 4.1, we get z^* is the KKT point of* (3.3)*, i.e., z^* is the nondominated solution of* (3.1)*.*

In the following, we give the numerical simulation for smart grid with controllable supply.

Example 1. we consider the social welfare of a single period of 24 time periods in the whole day, and there are 6 commercial users. We take $\omega = 4$, $d = 5$, $\rho = 0.55$, $\sigma = 0.4$, $\varepsilon = 10^{-4}$, The cost parameters are set as $a_k = 0.001$, $b_k = 0$, $c_k = 0$. All codes are run in Matlab Version R2018b. The results of this numerical simulation experiment are given by the following figures.

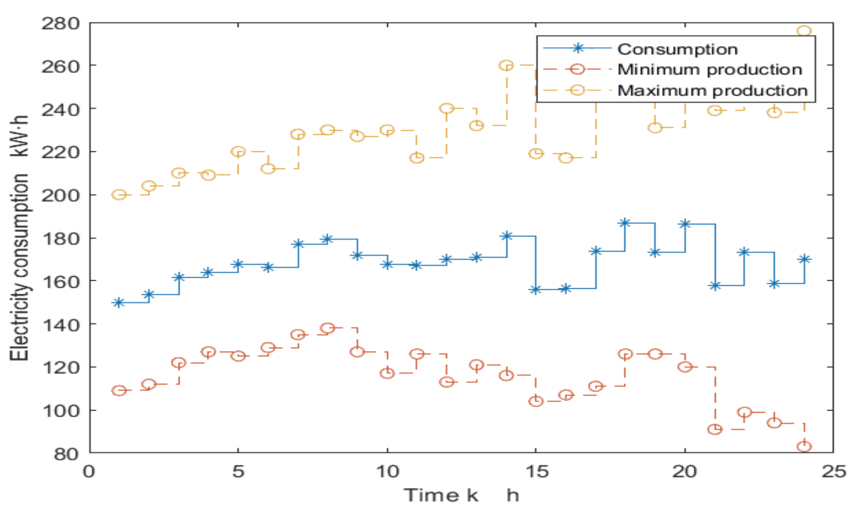

Figure 1: Electricity consumption

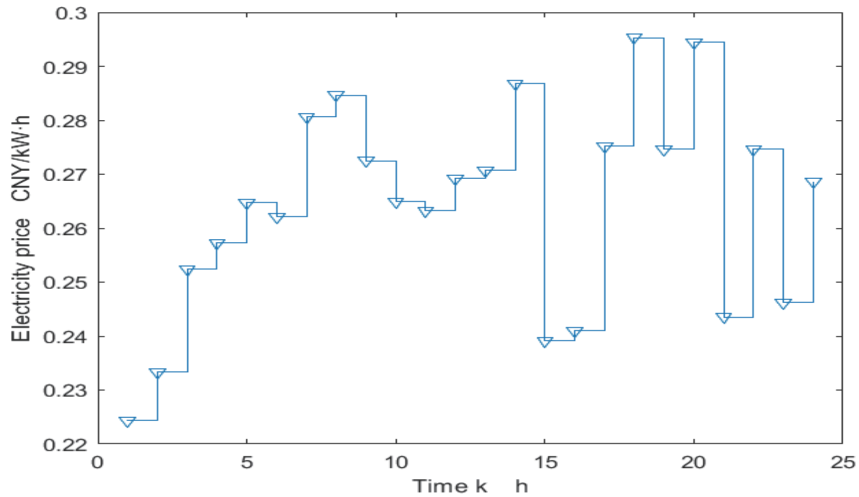

Figure 2: Optimal electricity price

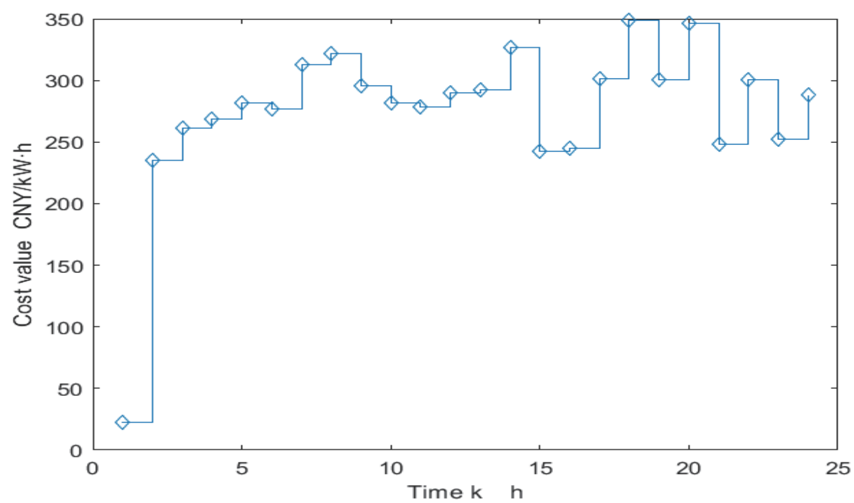

Figure 3: Cost of optimal electricity price

From the simulation results, we can see that data obtained meets the actual requirements. Algorithm 1 can solve the smart grid with controllable supply efficiently. The electricity consumption and optimal price are shown in Figure 1 and Figure 2. From Figure 1 we can see that the upper limit periods of the whole day are $8:00 \sim 9:00$, $14:00 \sim 15:00$, $18:00 \sim 19:00$, $20:00 \sim 21:00$ and the periods of low power consumption in the whole day are $1:00\sim 2:00$, $6:00\sim 7:00$, $15:00\sim 16:00$, $23:00\sim 24:00$. From Figure 2, we can see the real-time price for the time division k. We can see the cost, utility of electricity consumption and the social welfare value in Figure 3-5. From Figure 1-5, we can conclude that the smart grid can achieve balanced operation for the power grid through

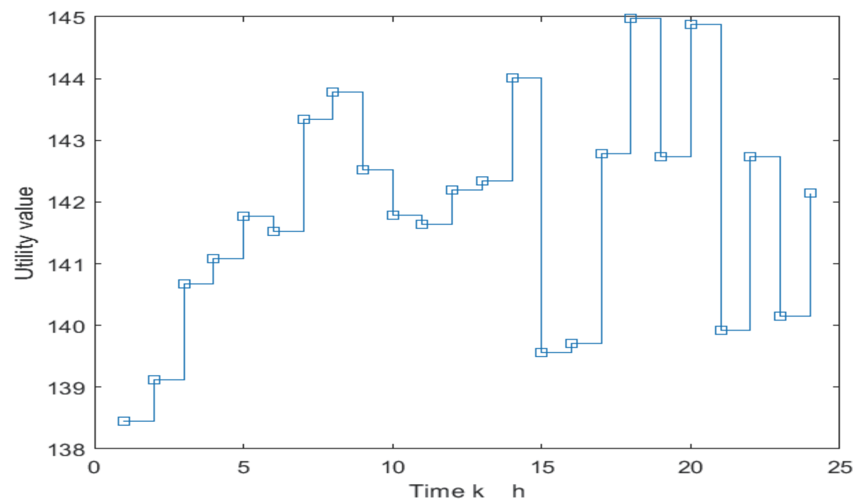

Figure 4: Utility of optimal electricity price

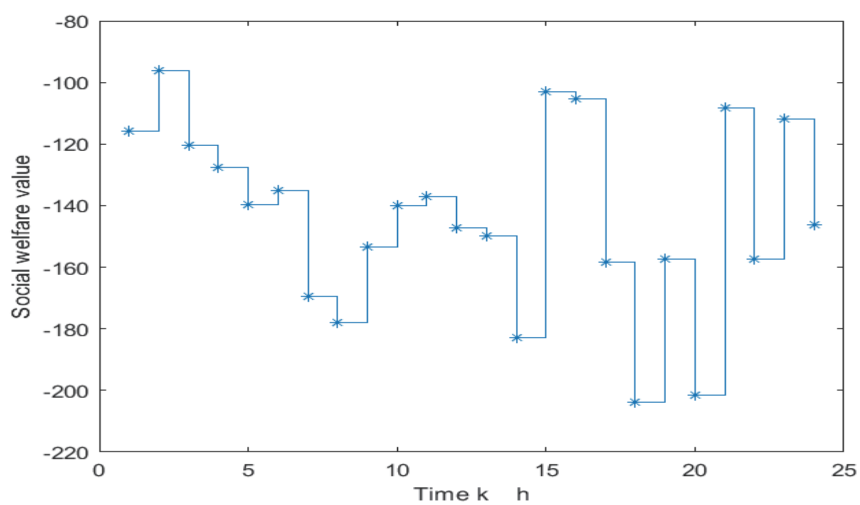

Figure 5: Social welfare value

timely adjustment and peak load shifting.

$\boxed{5}$ Conclusion

We propose the model of smart grid with controllable supply based on social welfare maximization firstly. Based on KKT conditions of the model, we transform it into semismooth equation system using NCP function, and then transform into an equivalent unconstrained optimization problem. We apply Levenberg-Marquardt method to solve it. From the numerical results, it can be seen that Levenberg-Marquardt method can effectively solve this problem. Our model can better reflect the real-time multi-user, power supply and demand relations. The research on this kind of problem further enriches the research work in the field of real-time price of smart grid based on maximizing social welfare, and it can be used to solve related optimization problems with interval conditions.

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